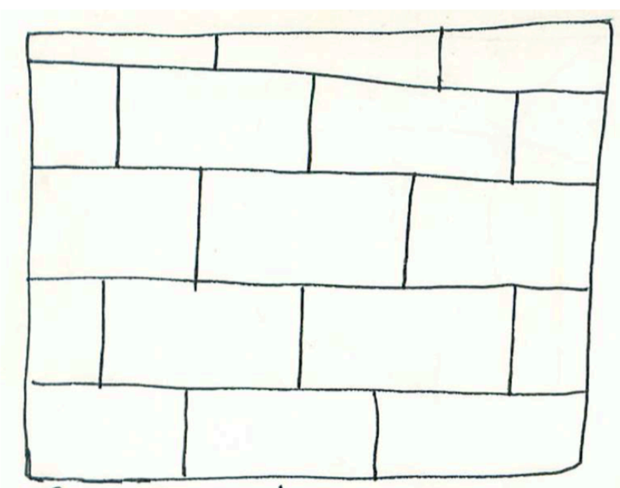
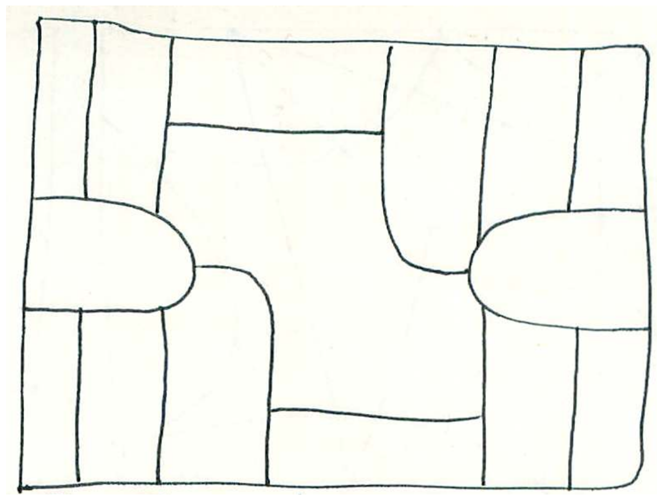
Map Coloring

We will call any division of part of a plane (e.g. a rectangle) into contiguous regions a *map*. We will define a *map coloring* to be a coloring of all the regions of a map so that each region contains a single color, and regions that share more than one boundary point have different colors.

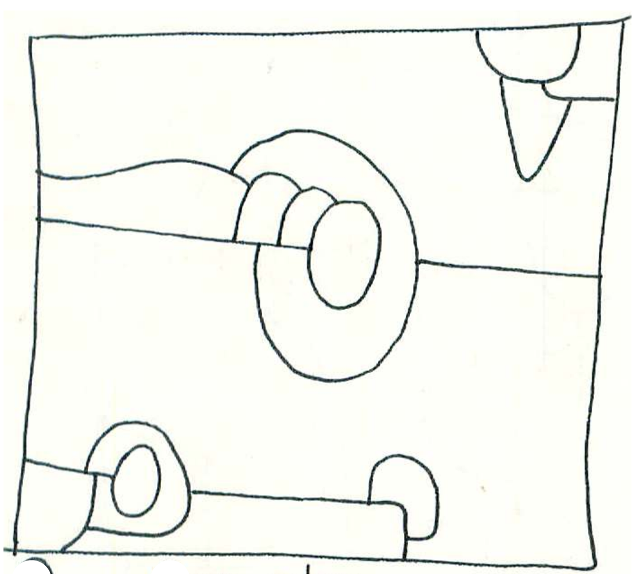
Mathematicians have long known that any map can be colored with four or fewer colors. The *four-color theorem* was proved in 1976 by analyzing hundreds of maps with the help of a computer.

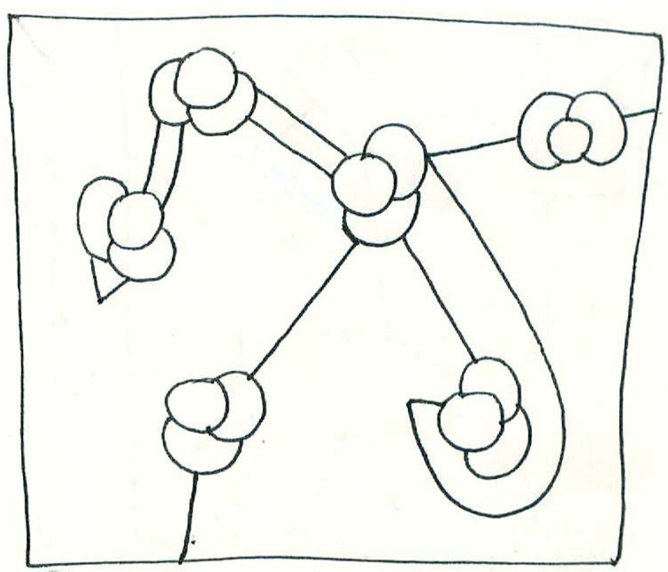
1. Color these maps with three colors:



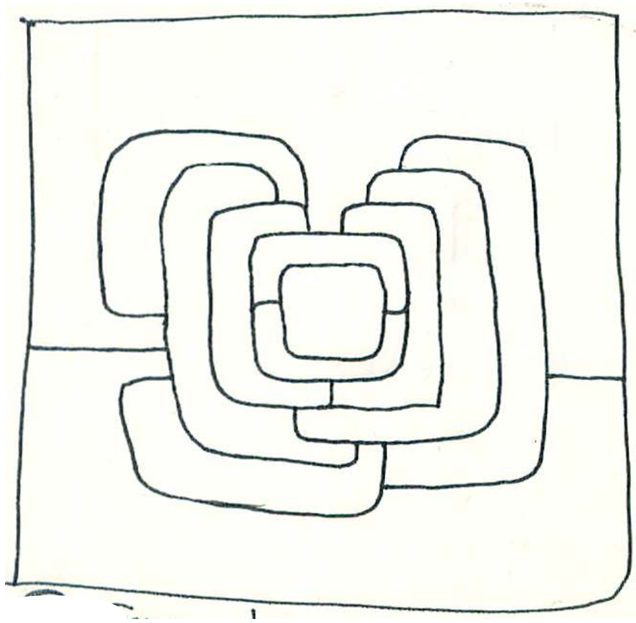


2. Color these maps with four colors:





3. Color this map with four colors:

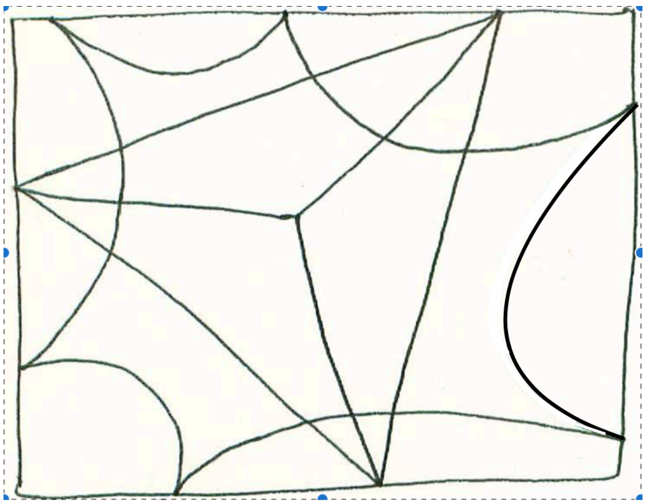
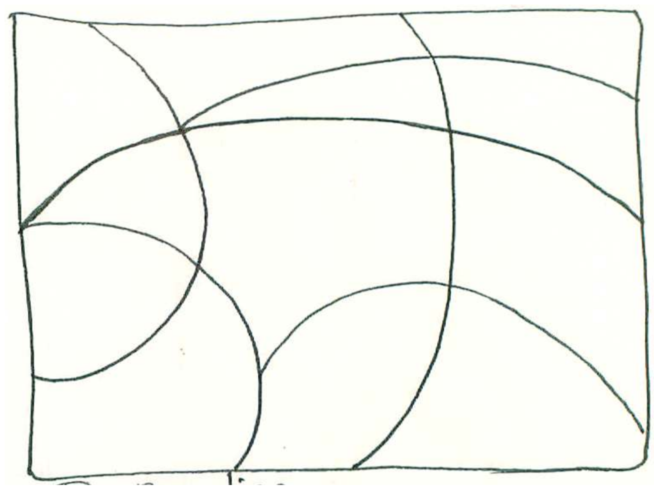


4. Make your own four-color maps. (In other words, maps that cannot be colored in two or three colors.)

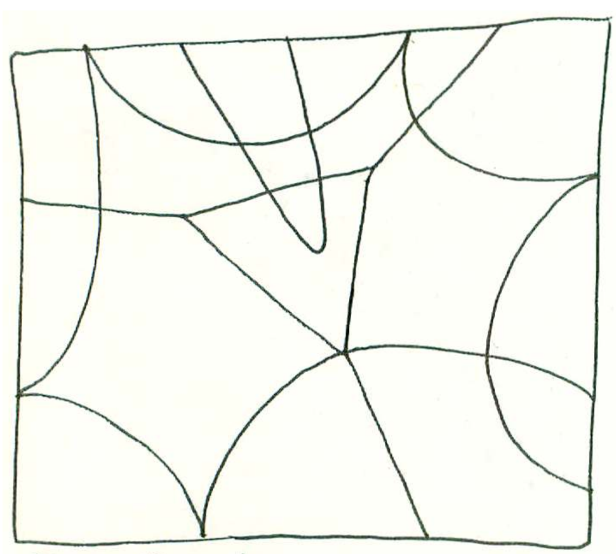
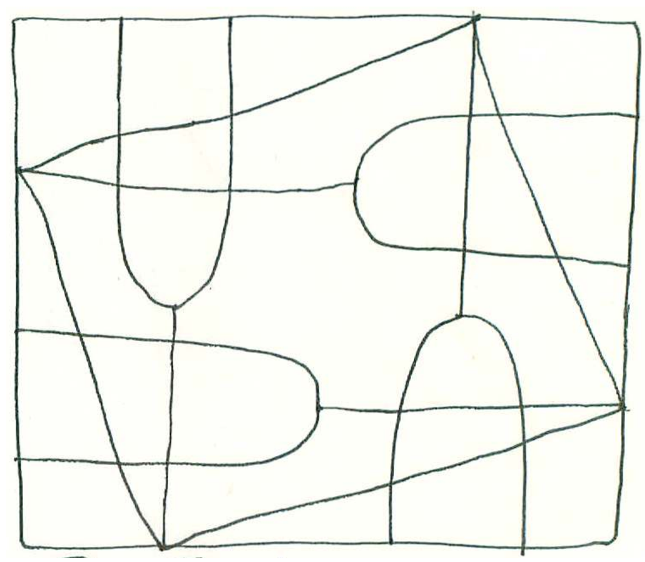
The Two-Color Theorem

In this activity, you will discover and state the *two-color theorem* by coloring maps, and trying to generalize what you observe.

1. These maps can be colored in two colors if you add a line in each one somewhere.
   Add the line and color each map using two colors only.

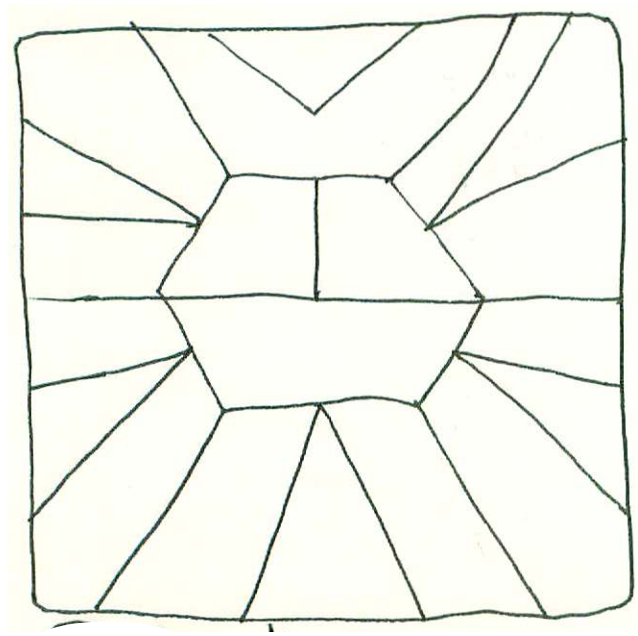



2. These maps can be colored in two colors if you add two lines in each.
   Add the lines and color each map using two colors only.

3. Think about what you needed to do in order to make two-color maps. Try to state what you learned as a theorem:
   A map is a two-color map if …
   (We may need to define some terms to state this clearly!)

4. What is the smallest number of lines that must be added to turn this into a two-color map? Explain.



5. Make your own two-color map puzzles.