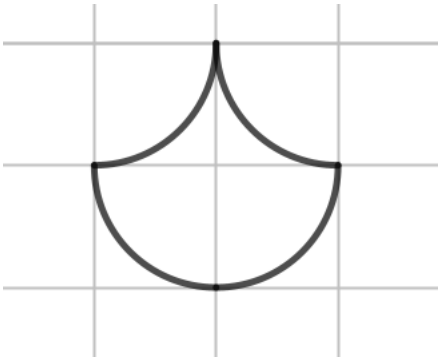
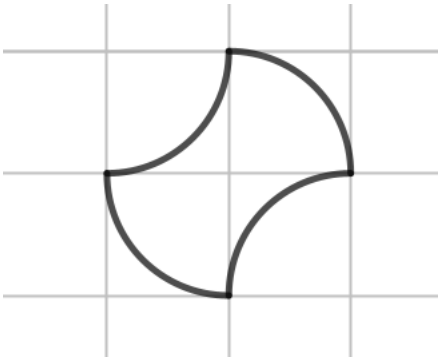
Leonardo's Areas

All of the figures below were created by Leonardo da Vinci.
Find the areas!

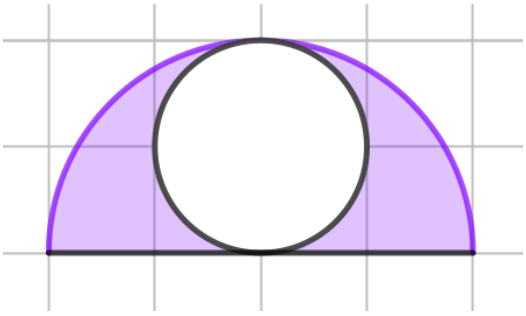
1. The Pendulum



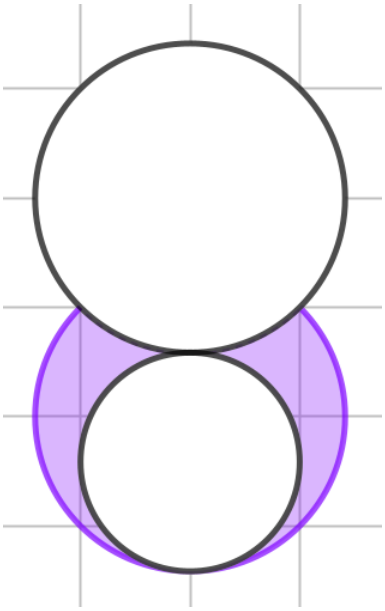
2. The Axe



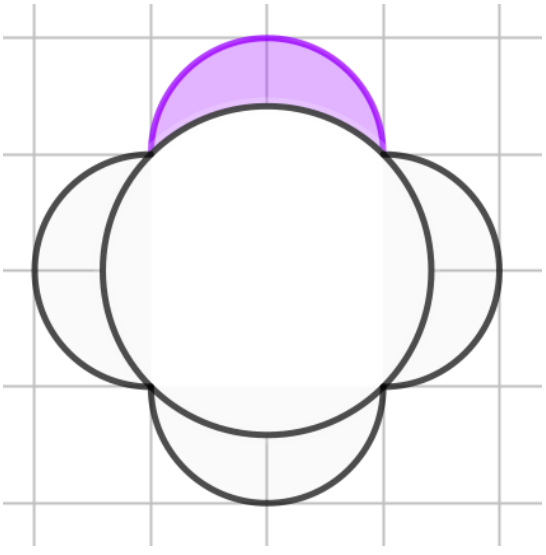
3. The Shark's Fin
(half of Cleopatra's Headdress)



4. The Claws



5. The Crescent



Teacher Notes

by Henri Picciotto

I found these figures in *Leonardo's Dessert: No π*, a booklet by Herbert Wills III, which contains many figures of this type. It was published by NCTM in 1985 and is now out of print. You can buy it from used-book sellers online, or borrow it from the Internet Archive.

You may ask students to consider each grid-paper unit to be 1, or r. In the notes below, I will use r.

Answers, Hints, and Connections

#1-2

Even though these figures' boundaries are curvilinear, their areas do not involve π — the answer is $2r^2$ in both cases. This can be seen readily by rearranging the pieces outlined by the grid paper.

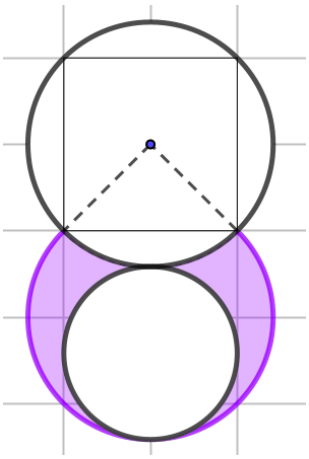
Note that both figures are polyarcs. See the discussion of polyarcs in my article (Geometric Puzzles in the Classroom), and in Meghan Lee's blog post (Polyarcs in the Classroom).

#3

The shaded area is equal to that of the unshaded circle: πr^2.

This illustrates an underlying fact: if you double the radius, you quadruple the area. I use that relationship between scaling factor and ratio of areas in a number of similar-figure activities in *Geometry Labs* and in various geometric puzzles. It is also relevant to the remaining two problems.

#4-5



This figure for #4 provides a crucial hint on how to proceed. By finding the circle radii, and then adding and subtracting pieces appropriately, students should find that each Claw's area is r^2. A similar strategy shows that the Crescent's area also is equal to that of one grid paper square, r^2. No π!

Wills's book presents solutions to these problems (and others) that do not require the formula for the area of a circle or the Pythagorean theorem, but those are complicated and not likely to be found by students. Still, you may enjoy looking them up.