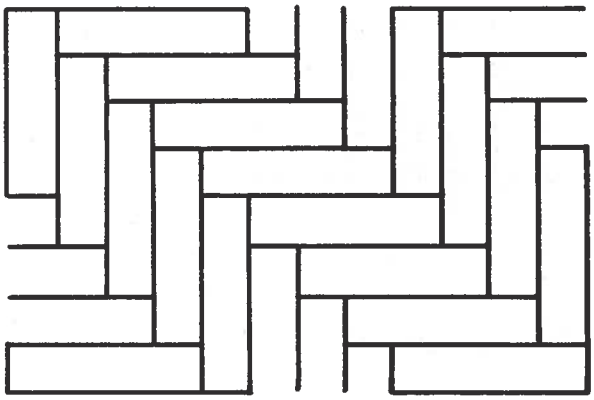
Tiling the Plane

Imagine a flat surface that extends forever in all directions. Such a surface is called a *plane*. If you had an unlimited supply of rectangular polyomino tiles, you could cover as large an area as you wanted. You could lay them out like this.

straight triomino tiling

This pattern could be extended in all directions. Other patterns are possible with rectangles. Here is one below.

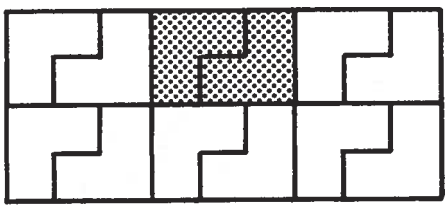


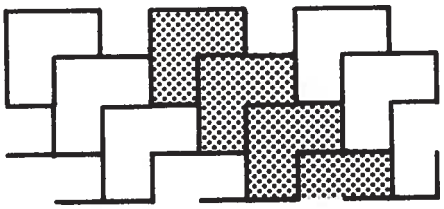
i tetromino tiling

Find interesting ways to tile a plane with the following four rectangular polyominoes. Use the grid below and some grid paper.

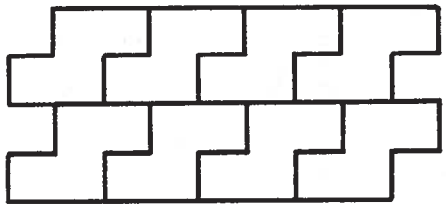
1. domino 2. square tetromino 3. I pentomino 4. 2-by-3 hexomino

Tiling with Other Polyominoes



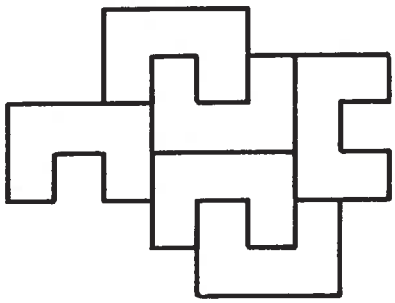


Here are 2 bent triomino tilings. They can be extended in all directions. The first one is made up of two-triomino rectangles. They can be used to tile the plane. The second one is made up of diagonal stripes. By making more stripes beside these, you can continue the pattern as far as you want.



1. Can you continue this n tetromino tiling in all directions? _____

   Use grid paper to show if you can do it.



2. Can you continue this U pentomino tiling in all directions? _____

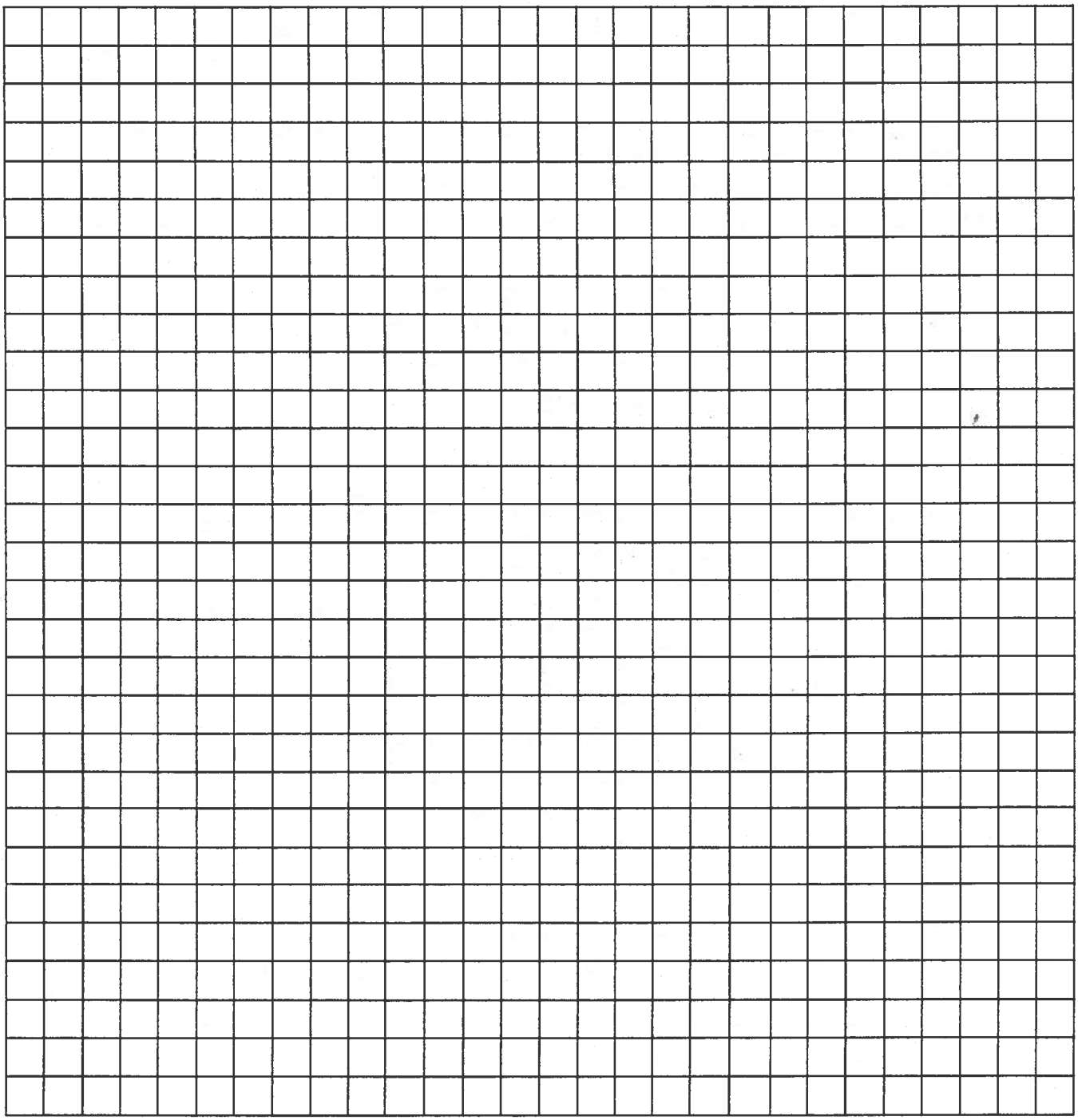
   Use grid paper. Tell what happens. _____

   _____

Tiling

Show one or more ways to tile with each of these polyominoes.

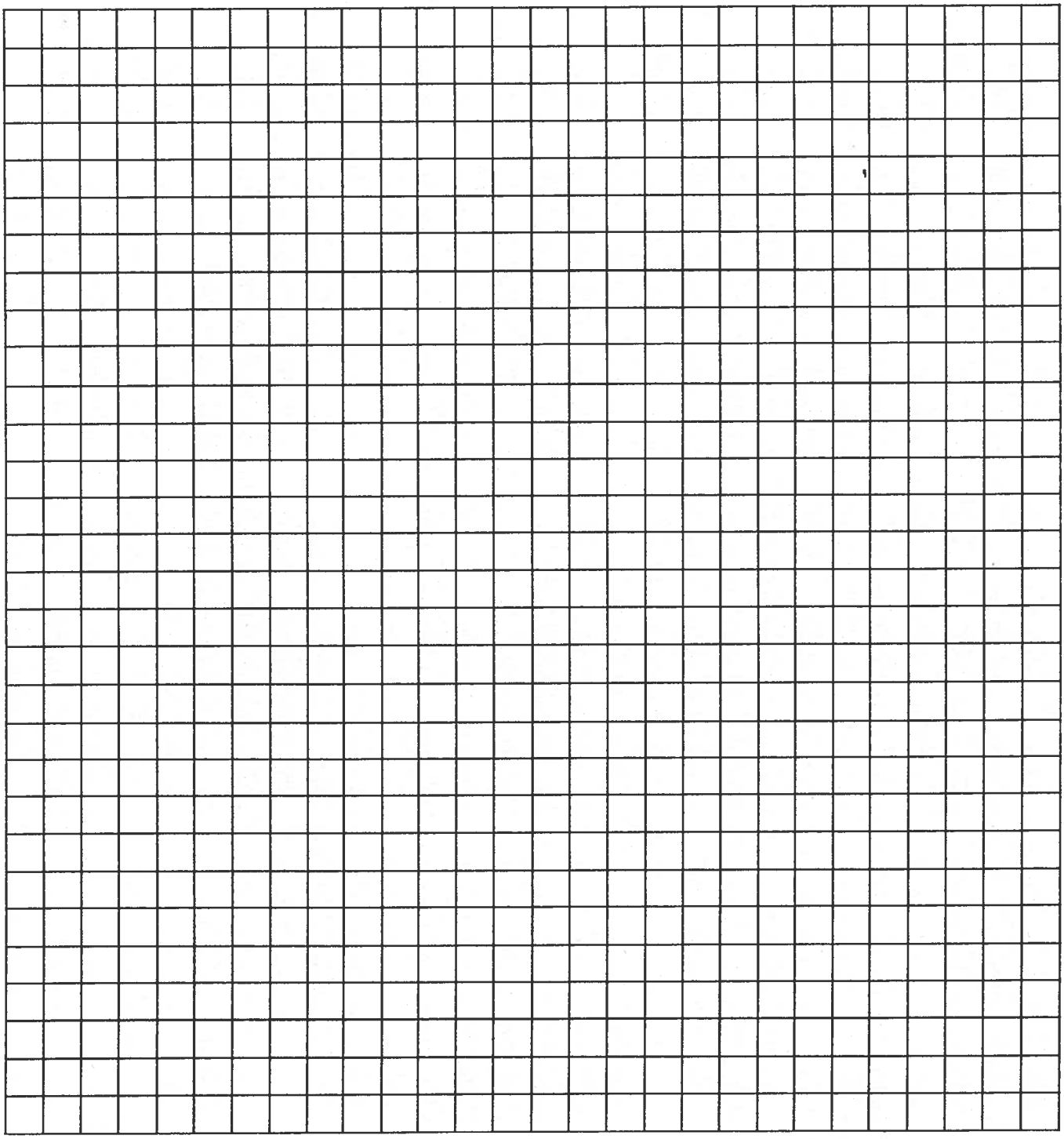
1. I tetromino
2. L pentomino
3. P pentomino
4. t tetromino



Tiling with Pentominoes

Show one or more ways to tile with each of these pentominoes.

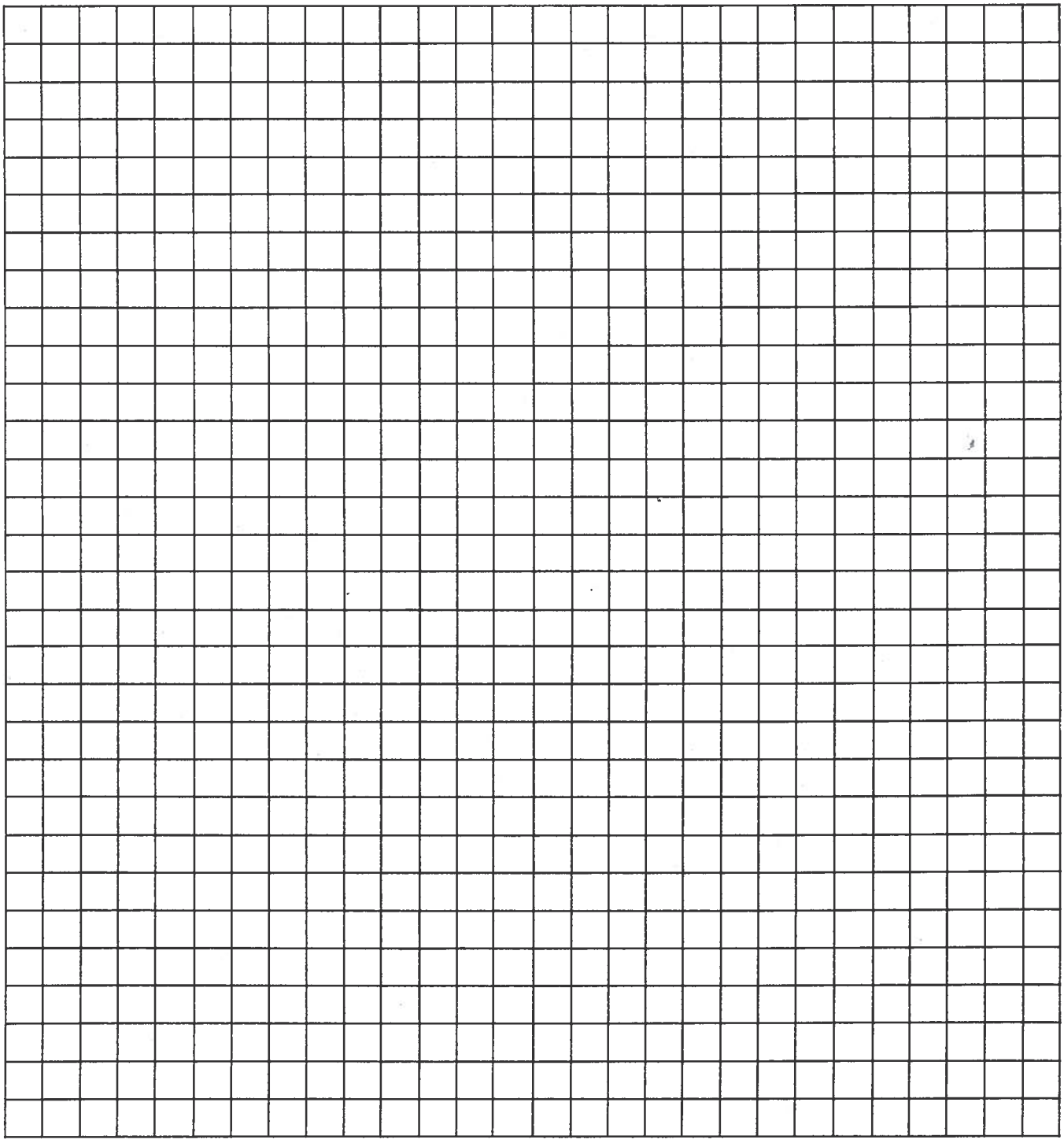
1. Y 2. F 3. N



Tiling with Pentominoes

For each pentomino, show one or more ways to tile.

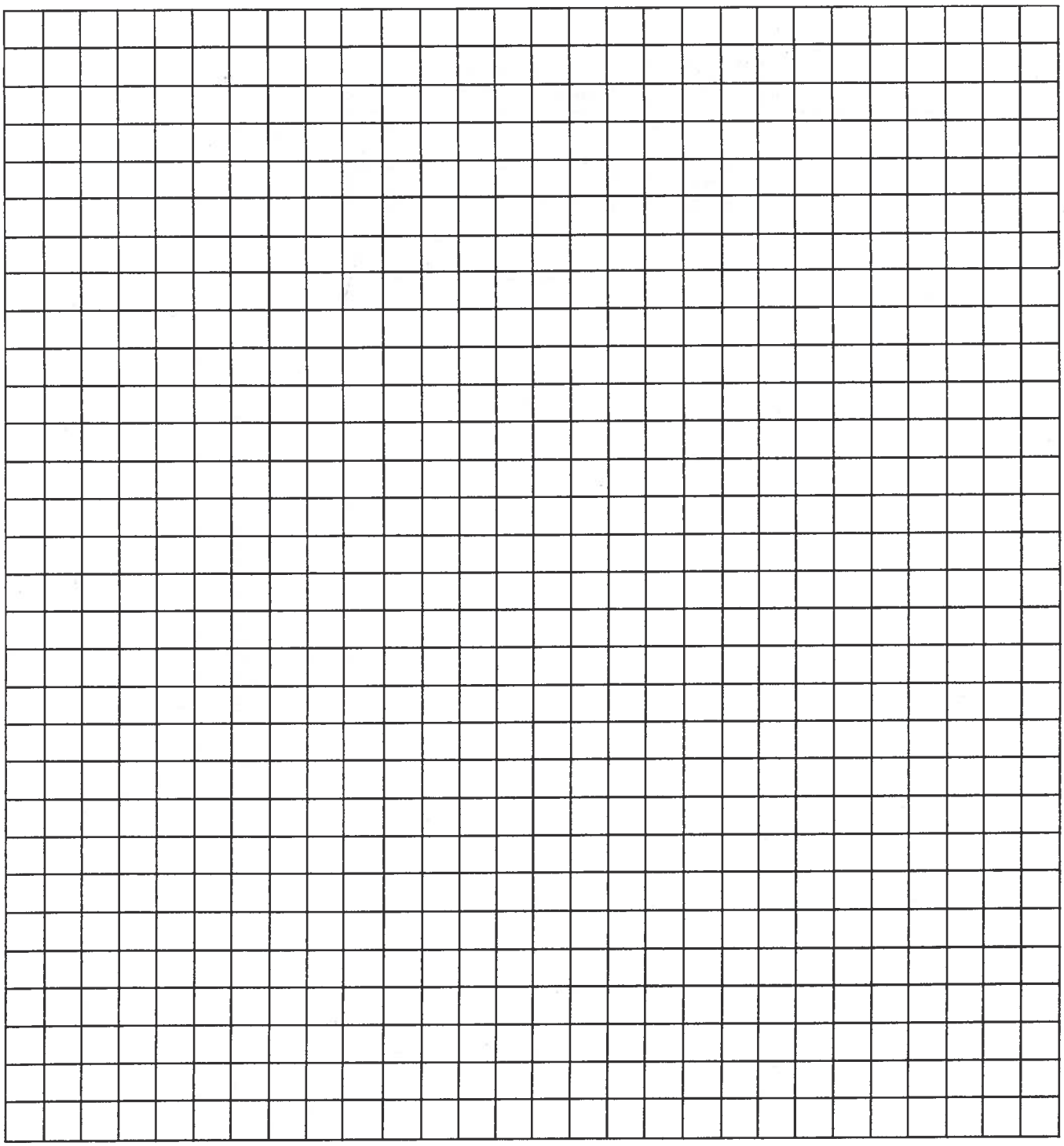
1. T 2. U 3. V



More Tiling with Pentominoes

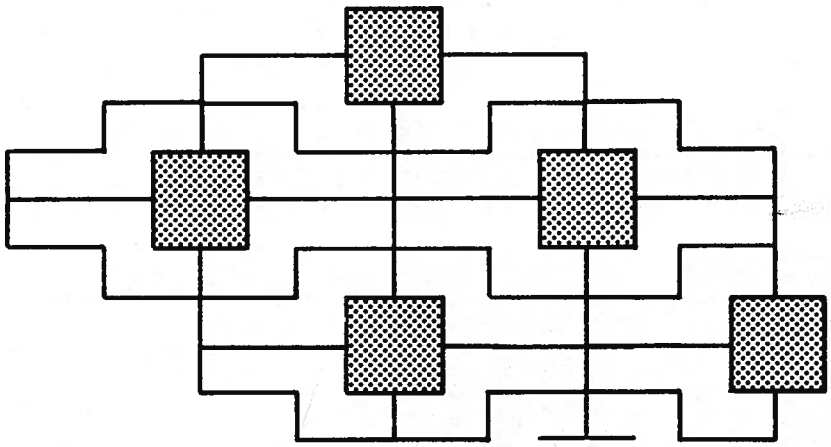
For each pentomino, show one or more ways to tile.

1. W
2. X
3. Z



Tiling with Two Polyominoes

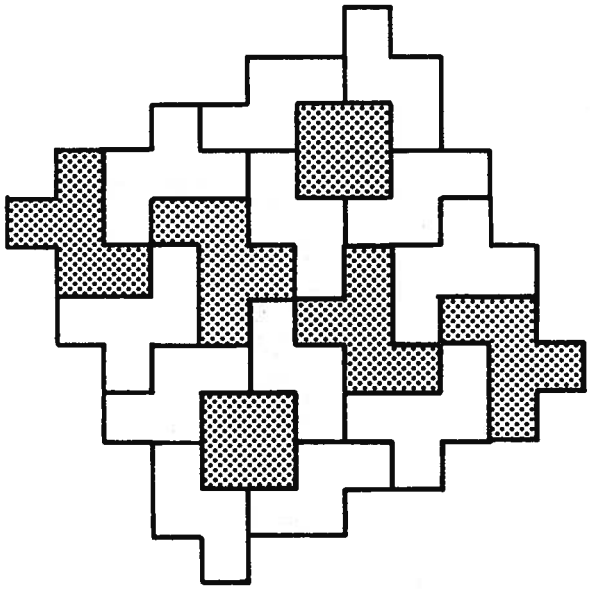
Here is a tiling that uses two polyominoes. Make up some more tilings of your own.



N and square

Tiling with Three Polyominoes

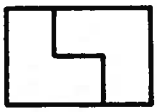
Here is a tiling that uses three polyominoes. Make up some more tilings of your own.



F, n, and square

Tiling Rectangles

If a rectangle can be covered exactly by copies of the same polyomino, then that polyomino tiles the rectangle. For example, here is the smallest rectangle that can be tiled by the bent triomino.



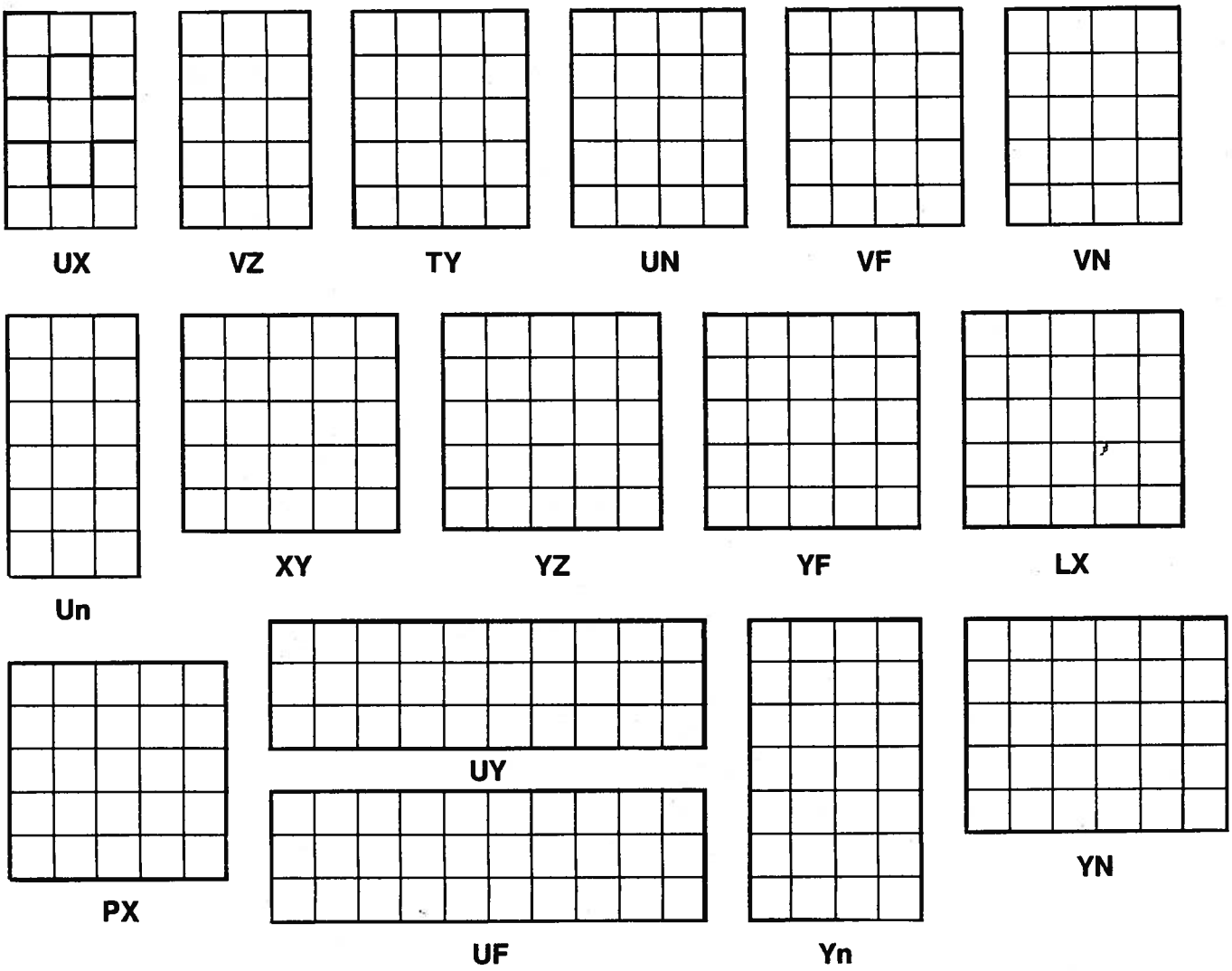
2 by 3

For each of the polyominoes listed below, find the smallest rectangle that it tiles. Show your solutions on the grid.

1. Tetrominoes: I, n, t ★
2. Pentominoes: L, P, Y ★

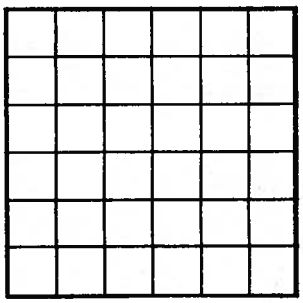
Tiling with Polyomino Pairs

All these squares and rectangles can be tiled by pairs of polyominoes. For example, you can tile the 3-by-5 rectangle with two U's and one X. Show how to cover each shape below using the given polyominoes.

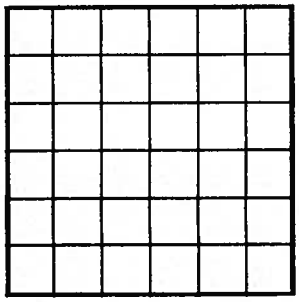


More Tiling with Polyominos

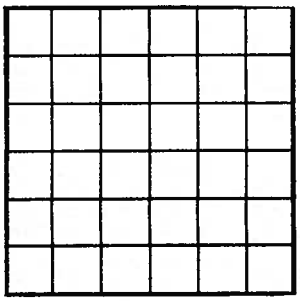
Now tile these squares and rectangles with pairs of polyominoes. Show how you covered each shape by using the given polyominoes.



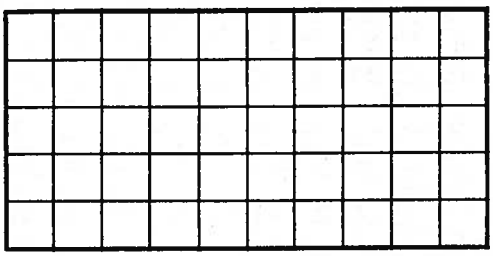
Tn



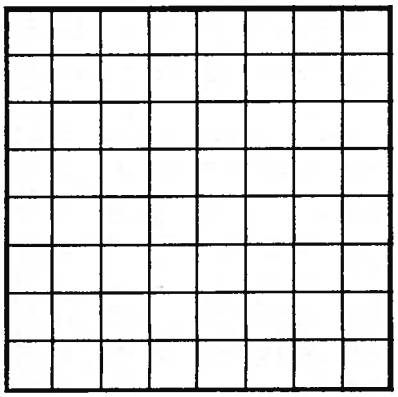
Yn



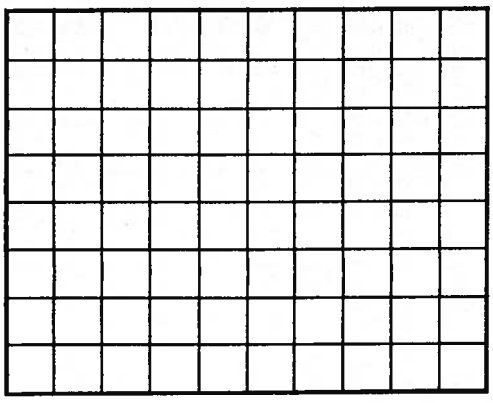
Pn



TN



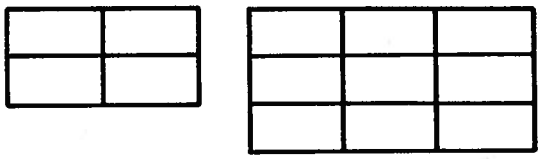
Ln



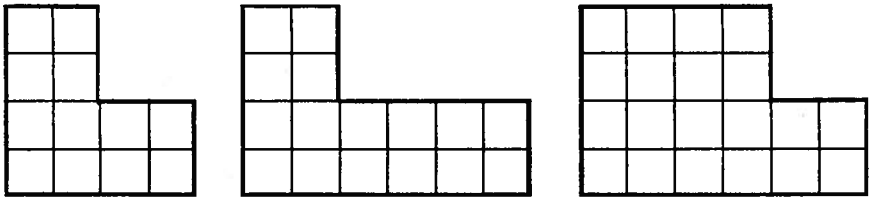
TW

Rep-tiles

A shape is a rep-tile if it can be used to tile a larger copy of itself. For example, a domino is a 1-by-2 rectangle. If you double its dimensions, you get a 2-by-4 rectangle. If you triple them, you get a 3-by-6 rectangle. These larger copies of the domino can be tiled by dominoes. Therefore the domino is a rep-tile.



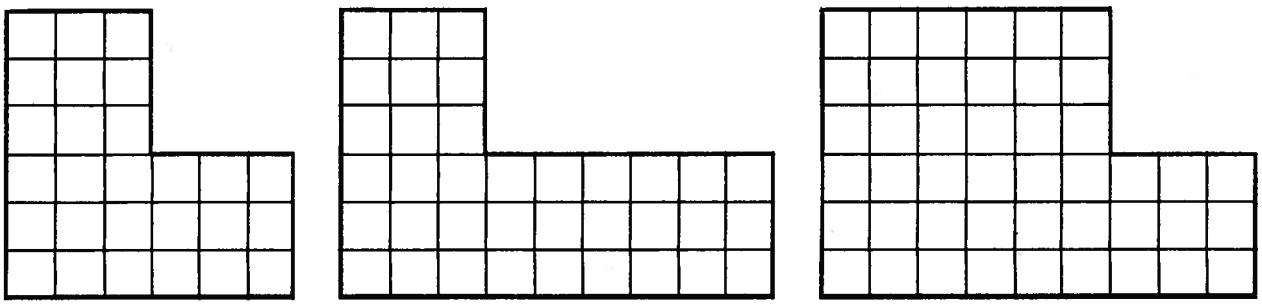
1. The bent triomino, the I tetromino, and the P pentomino are rep-tiles. Here are these polyominoes with their dimensions doubled. Tile the blown-up version of each one with the original shape.



2. How many original shapes did you need to cover each doubled figure?_____

3. When the dimensions of a figure are doubled, its area is multiplied by_____.

4. Here are the bent triomino, the I tetromino, and the P pentomino with their dimensions tripled. Tile the blown-up version of each one with the original shape.

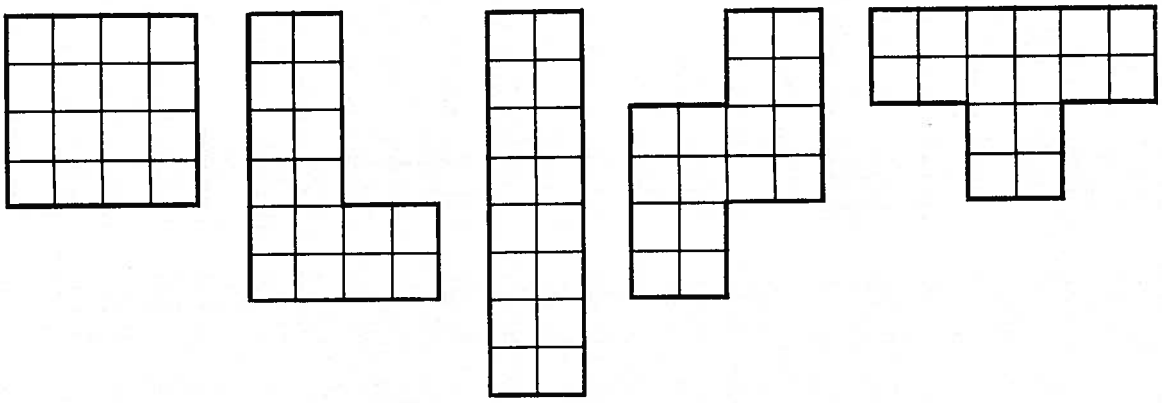


5. How many original shapes did you need to cover each tripled figure?_____

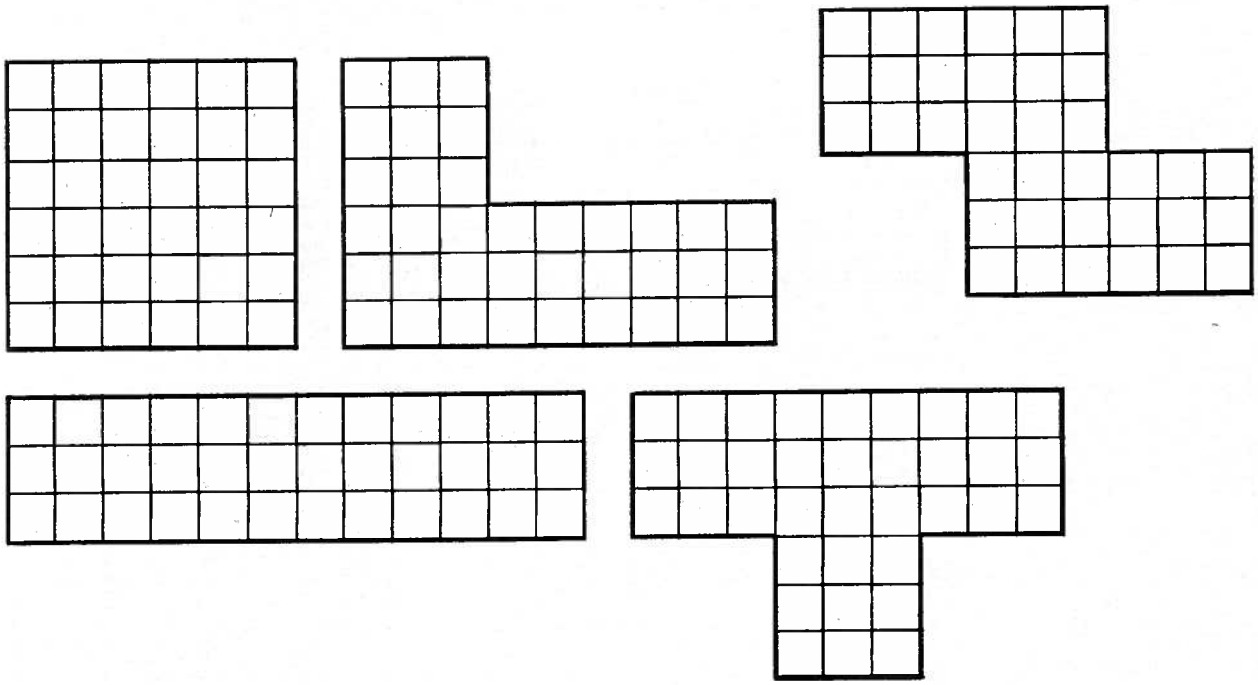
6. When the dimensions of a figure are tripled, its area is multiplied by _____.

Doubled and Tripled Tetrominoes

1. Tile this set of doubled tetrominoes using only the l tetromino.

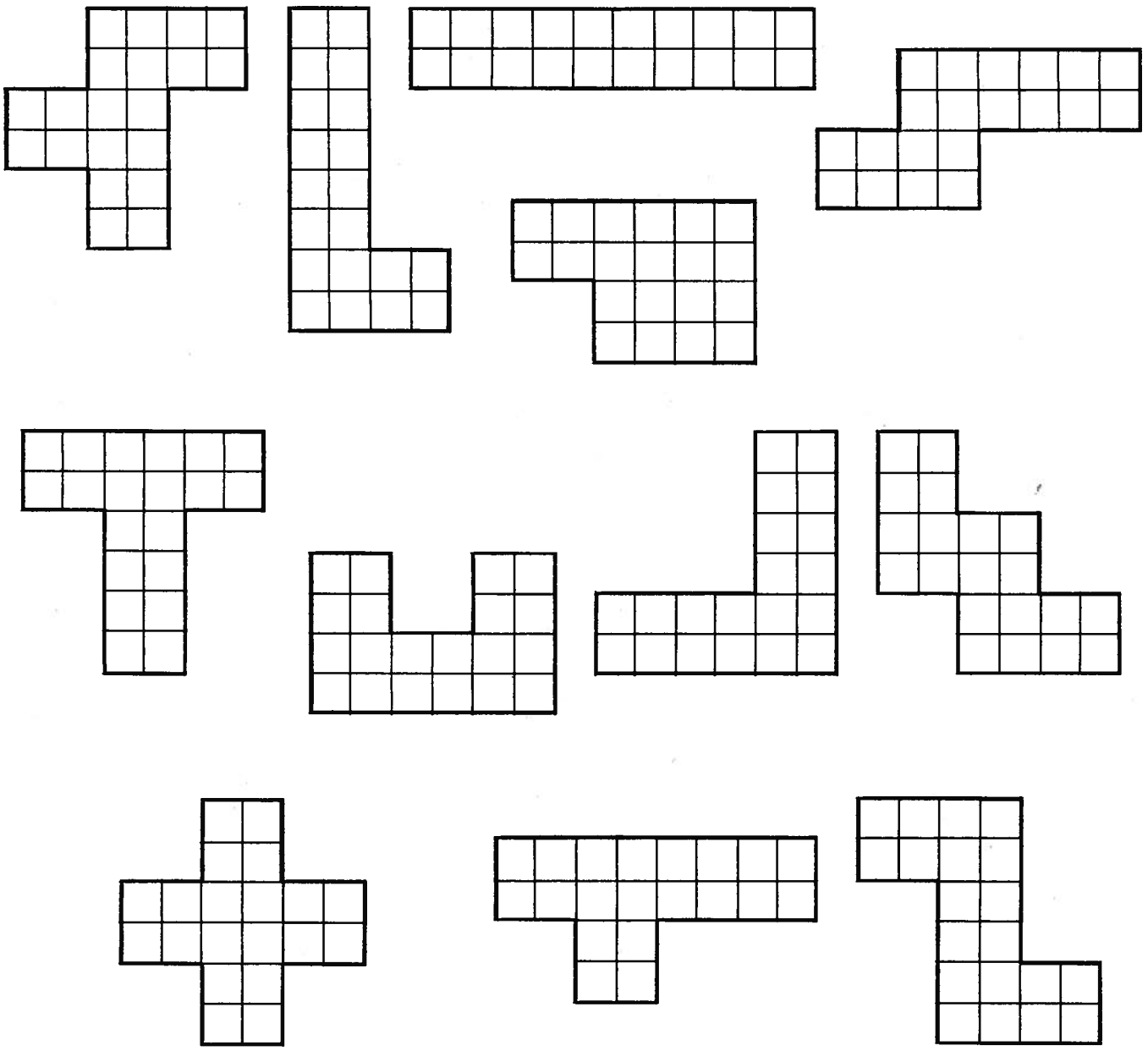


2. Now tile this set of tripled tetrominoes using just the l and t tetrominoes.



Doubled Pentominoes

Tile this set of doubled pentominoes using only the P and N pentominoes.
(You need only three N's.) ★



Tripled Pentominoes

Tile this set of tripled pentominoes using only the P and L pentominoes.