Geometric Puzzles
Virtually
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Why Puzzles?

◊ Fun and interesting!
◊ Works with all ages
◊ Status equalizer
◊ Math content and practices
Virtual Manipulatives

vs. concrete materials
Tangrams
Line symmetric

Not line symmetric
◊ Make a geometric figure, using 1 to 7 pieces

◊ Take a screenshot

◊ Share on the appropriate page, indicating the number of pieces

◊ Add pages if needed
What squares are possible?
Tangram Measurements

(inches and square inches)
<table>
<thead>
<tr>
<th>Pieces</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Side</td>
<td>$\sqrt{2}$</td>
<td>$\sqrt{2}$</td>
<td>2</td>
<td>$2\sqrt{2}$</td>
<td>$2\sqrt{2}$</td>
<td>4</td>
</tr>
</tbody>
</table>

$\sqrt{8} = 2\sqrt{2}$
A 6-piece square is impossible

◊ Total tangram area: 16 in²
◊ 6-piece area: 15, 14, or 12 in²
◊ Cannot be a tangram square!
Convex Polygons

All angles < 180°
What convex tangram figures are possible?
triangles, quadrilaterals, pentagons, …?
Exterior Angles

4 \times 90^\circ = 360^\circ

135^\circ + 135^\circ + 90^\circ = 360^\circ

(turn angles)
A convex 9-gon is impossible  
(proof by zombie)

◊ All tangram angles are multiples of 45°  
◊ Greatest possible interior angle: 135°  
◊ Least possible exterior (turn) angle: 45°  
◊ $8 \times 45° = 360°$ so there cannot be 9 angles

A convex 8-gon is impossible  
(proof by trying to do it!)
← convex 7-gon
Polyominoes are shapes that are made by joining squares edge-to-edge.
Pentominoes
Holes

8 holes. Can you get more?
Congruent Figures

Triples?
Pentomino Rectangles
Pentomino Rectangles

◊ If a figure can be covered with pentominoes, what can you say about its area?

◊ If a figure can be covered with pentominoes, what can you say about its sides?

◊ If a rectangle can be covered with pentominoes, what can you say about its sides?
Pentomino Blowups

Double the dimensions (two cannot be solved)
Triple the dimensions (all are possible, but hard!)
◊ When the dimensions are doubled, the area is multiplied by 4

◊ When the dimensions are tripled, the area is multiplied by 9
◊ Ratio of areas?

◊ Scaling factor?
Taking it further!

◊ Rep-tiles

◊ Supertangrams
Lots of links in the “handout” on my **Talks** page.

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