Geometric Puzzles: Tiles and Rep-Tiles

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Tangrams
Make a square, using 1 to 7 pieces.

What squares are possible?
Tangram Measurements
(inches and square inches)
<table>
<thead>
<tr>
<th>Pieces</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Side</td>
<td>$\sqrt{2}$</td>
<td>$\sqrt{2}$</td>
<td>2</td>
<td>$2\sqrt{2}$</td>
<td>$2\sqrt{2}$</td>
<td>4</td>
</tr>
</tbody>
</table>

$\sqrt{8} = 2\sqrt{2}$
A 6-piece square is impossible

◊ Total tangram area: 16 in$^2$
◊ Individual pieces: 1, 2, or 4 in$^2$
◊ 6-piece area: 15, 14, or 12 in$^2$
◊ Side of 6-piece square?
Convex Polygons

All “turn angles” turn in the same direction

Yes

Yes

Yes

No

All interior angles < 180°
What convex tangram n-gons are possible?

triangles, quadrilaterals, pentagons, …?
Exterior Angles

4 \times 90^\circ = 360^\circ

135^\circ + 135^\circ + 90^\circ = 360^\circ

(turn angles)
A convex 9-gon is impossible (proof by zombie)

◊ All tangram angles are multiples of 45°
◊ Greatest possible interior angle: 135°
◊ Least possible exterior (turn) angle: 45°
◊ $8 \times 45° = 360°$ so there cannot be 9 angles

5-gon? 6-gon? 7-gon? 8-gon?
← convex 5-gon

← convex 6-gon

← convex 7-gon
← convex 8-gon? 😞
Polyominoes

(closed grid-paper figures: no diagonals, no crossings)
In-corners and out-corners

What is the pattern?
Why is it always true?
Proof by zombie again!

Total turning: $4 \times 90^\circ = 360^\circ$

Every additional right turn must be canceled by a left turn.
Pentominoes
Holes
Three-piece pentomino puzzles

Pentomino Puzzles

Pentominoes
Layers
(congruent figures)
Generalizing

◊ What rectangles are possible?
◊ What “triangles” are possible?
◊ What simultaneous rectangles?
◊ …triangles? combinations?
Pentomino Blowups

double the dimensions
(two cannot be solved)

triple the dimensions
♦ When the dimensions are doubled, the area is multiplied by 4

♦ When the dimensions are tripled, the area is multiplied by 9
half-square

no!

supertangram = four half-squares, joined edge-to-edge

Find them all!
Rep-tiles
Chair tiling
Find some rep-tiles!

use the template, grid paper, and/or triangle paper
Rep-Triangles

Find triangles that can be tiled with 2, 3, 4, 5, ... scaled copies of themselves.
initial
Pinwheel tiling
Reminder:
Lots of links in the “handout” on my Talks page.

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