Space

An Alternate Elective after Algebra II

Henri Picciotto

MathEducation.page
Math on Another Planet

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\]
A Long Month

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| 33  | 34 | 35 | 36 | 37 | 38 |    | ...

[...]
### Isomorphism

#### Algebra: Themes, Tools, Concepts

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**Abstract Algebra p. 12**

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Space: topics

- Abstract algebra

◊ Transformational geometry

◊ Symmetry

◊ Dimension

- 3D: polyhedra

- 4D: introduction
Transformations

◊ Transformations: one-to-one functions (domain, range: the whole plane)

◊ Isometries: transformations that preserve distance
Fundamental Theorem of Isometries: every isometry of the plane is a reflection, a rotation, a translation, or a glide reflection.
Transformations

Symmetry

Dimension

Computing transformations using complex numbers:

◊ Translation: add $a+bi$

◊ Rotation around the origin: multiply by $\cos \theta + i \sin \theta$

◊ Rotation around $(a,b)$: subtract $a+bi$, rotate around the origin, add $a+bi$
Computing transformations using matrices
Transformations
Symmetry
Dimension
The Seven Line Symmetry Groups

1

2

3

4

5

6

7
Transformations
Symmetry
Dimension
Figure 18.7
(a) Victorian ornament
(b) Arabian
(c) Pompeian mosaic
(d) Byzantine mosaic
Transformations
Symmetry
Dimension
Transformations
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Transformations
Symmetry
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Transformations
Symmetry
Dimension
Transformations
Symmetry
Dimension: 3D
◊ Platonic and Archimedean polyhedra

◊ Duality

◊ Euler’s and Descartes’ theorems

◊ Review of geometry and trigonometry
Transformations
Symmetry
Dimension: 3D

The chief reason for studying regular polyhedra is still the same as in the time of the Pythagoreans, namely, that their symmetrical shapes appeal to one's artistic sense.

---H.S.M. Coxeter
Transformations
Symmetry
Dimension: 3D
Transformations
Symmetry
Dimension: 3D
Transformations
Symmetry
Dimension: 3D
Transformations
Symmetry
Dimension: 3D
Transformations
Symmetry
Dimension: 4D
LAB 6.5

Slicing a Cube

**Equipment:** Transparency, stiff paper, scissors, adhesive tape

Imagine that you slice a cube in a direction parallel to one of the faces. The shape of the slice will be a square. Now imagine that you slice a cube parallel to one of the bottom edges, but tilted from the horizontal. The shape of the slice will be a rectangle. What shapes are possible for a slice? To investigate this, you will use a hollow transparent cube and stiff paper to simulate the slices.
Transformations
Symmetry
Dimension: 4D
Transformations
Symmetry
Dimension: 4D
Transformations

Symmetry

Dimension: 4D
Transformations
Symmetry
Dimension: 4D
Space
An Alternate Elective after Algebra II

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Urban School

Math 1

More accessible

Programming

Math 2

Math 3

More Challenging

MMaPS

 AMA

Functions

Analytic Geometry

Infinity

Space

Calculus

AP Calc AB

AP Calc BC

two terms  one term half a term
Space overview

Who takes the class

Topics
- Juniors, before Calculus

Review
- Seniors, instead of or in addition to Calculus

Resources

Computer tools
Space overview

Who takes the class

Topics
- Abstract algebra
- Transformations
- Symmetry
- Dimension (3D, 4D)

Review

Resources

Computer tools
Space overview

Who takes the class

Topics

Review

Resources

Computer tools

Algebra

Geometry

Trigonometry
Space overview

Who takes the class

Transformational Geometry
by Richard Brown

Topics

Algebra: Themes, Tools, Concepts
by Anita Wah and Henri Picciotto

Review

Geometry Labs
by Henri Picciotto

Resources

Handbook of Regular Patterns
by Peter Stevens

Computer tools

Zome Geometry
by George Hart and Henri Picciotto

Flatland
by Edwin Abbott
Space overview

Who takes the class

Topics

Review

Resources

Computer tools

Cabri II+

Cabri 3D

(vZome)
Space
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