Practical Strategies

to reach the full range of students

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Reaching All Students

"The California Mathematics Council (CMC) believes that **all students** have the capacity to become mathematically competent and confident when provided a rigorous and challenging mathematical program supported by high expectations."

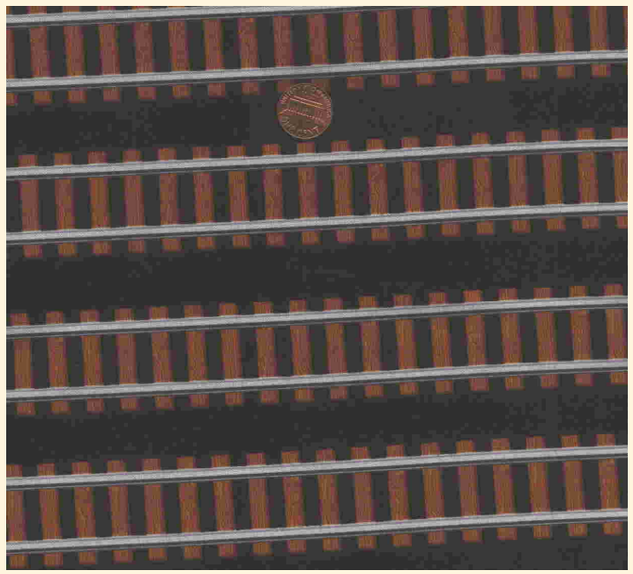
However…

Students learn math concepts
at different rates.

All classes are heterogeneous.

Flawed Responses

1. Tracking



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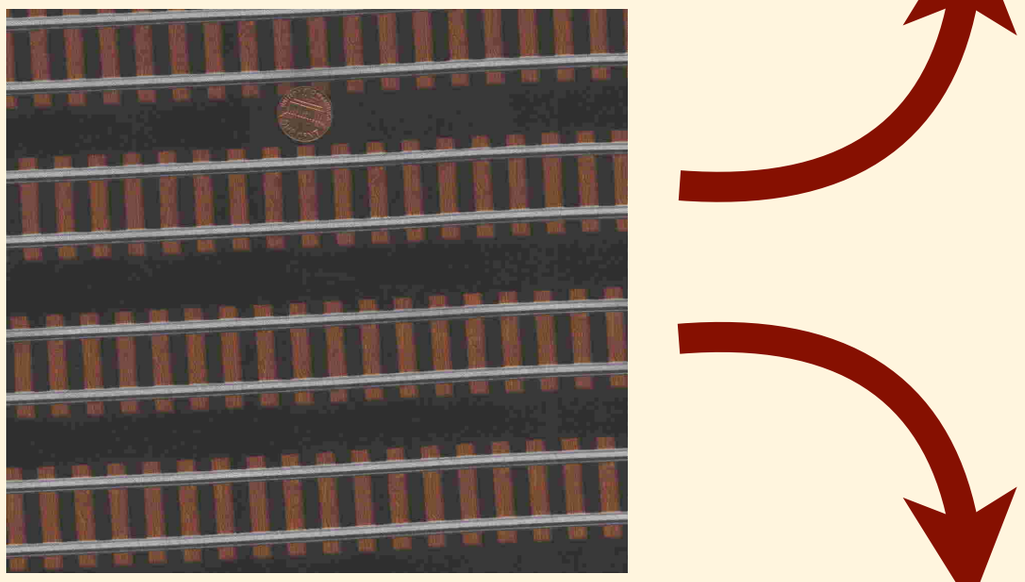

2. Acceleration

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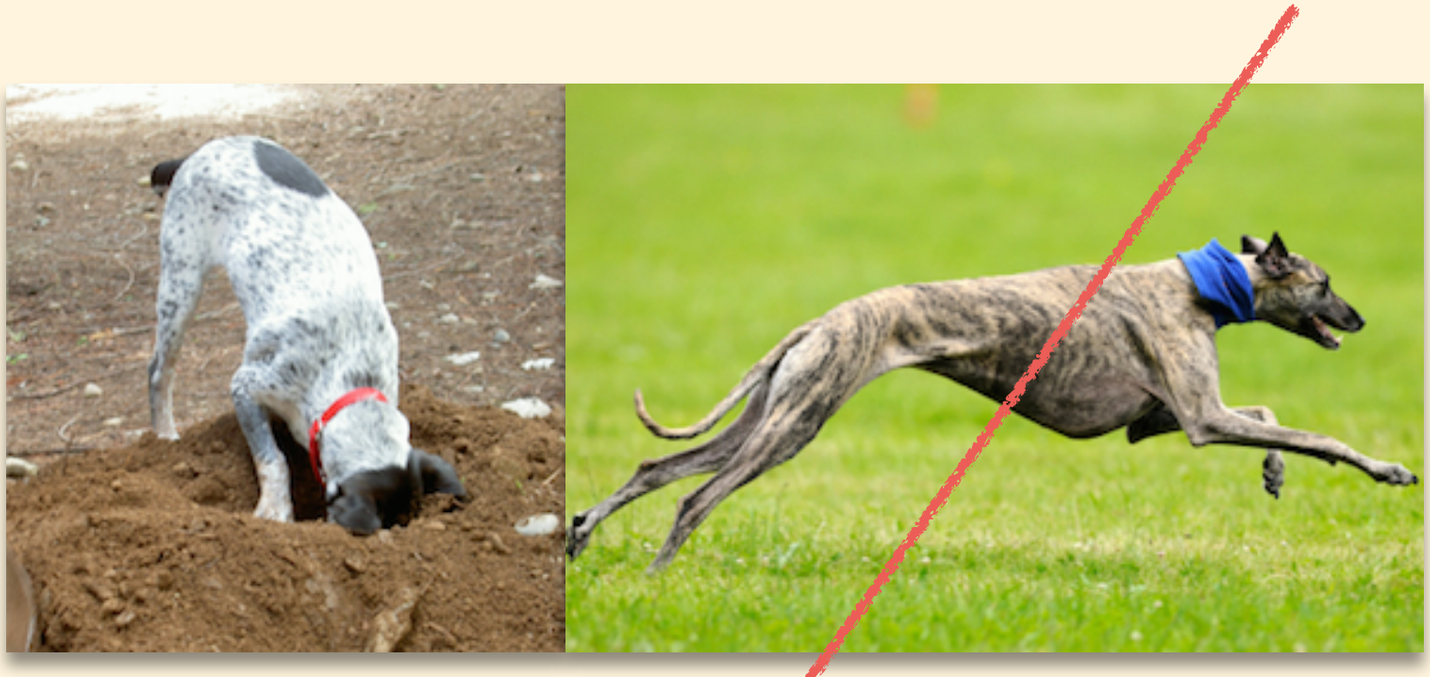
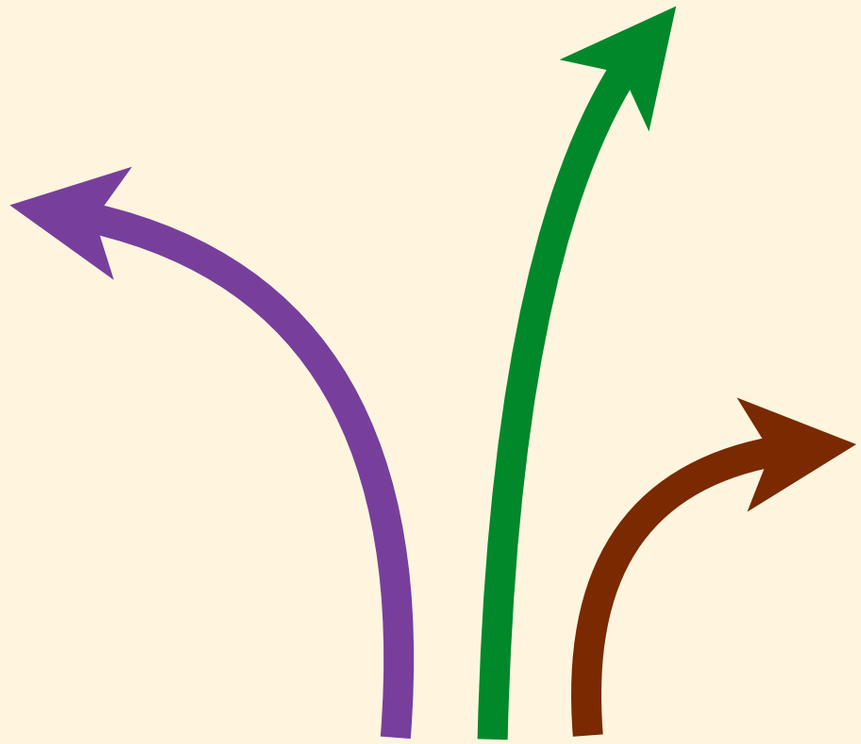
OK, within reason (one year)



For understanding and retention

Go Deeper, Not Faster

3. Differentiation
(aka individualization)



◊ Destroys community of learners

◊ Too much work for the teacher

Many good ideas spread through…

Many good ideas spread through…

Fads

Many good ideas spread through…

Fads

but…

Nothing Works!

for every student,
every topic,
every day,
every class,
every school,
etc...

We must be eclectic!

(Select “what appears to be best in various doctrines”)

(Merriam-Webster)

Reaching the Full Range

Underlying Principles

Students must engage intellectually.

We need:

Access *and* challenge

Support for all

Alliance with the strongest

Importance of strong students

◊ Politically

◊ Philosophically

◊ Pedagogically

Reaching the Full Range

◊ differentiate by time, not content

◊ problem-centered curriculum

◊ group work

◊ whole-class discussion techniques

◊ tool-rich pedagogy

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Teacher, Know Yourself

First step:
differentiate by time, not content

Time pressure

is bad for students

Pacing

◊ Constant forward motion

◊ Eternal review

Practical Strategies

Extend Exposure while moving forward

Slow Down

each unit

Slow Down

each unit

Lag homework

Week 1: topic 1

Week 2: homework 1

Week 2: topic 2

Week 3: homework 2

Slow Down

each unit

Lag homework

Week 1: topic 1

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Slow Down

each unit

Lag homework

Week 1: topic 1

Week 2: homework 1

Week 2: topic 2

Week 3: homework 2

Slow Down

each unit

Lag assessments

Week 1: topic 1

Week 2: homework 1

Week 2: topic 2

Week 3: quiz 1

Week 3: homework 2

Week 4: corrections 1

Week 4: quiz 2

Week 5: corrections 2

Slow Down

each unit

Lag assessments

Week 1: topic 1

Week 2: homework 1

Week 3: quiz 1

Week 4: corrections 1

Week 2: topic 2

Week 3: homework 2

Week 4: quiz 2

Week 5: corrections 2

Slow Down

each unit

Lag assessments

Week 1: topic 1

Week 2: homework 1

Week 3: quiz 1

Week 4: corrections 1

Week 2: topic 2

Week 3: homework 2

Week 4: quiz 2

Week 5: corrections 2

Built-in review!

Slow Down

quizzes and tests

Slow Down

quizzes and tests

Give students as much time as they need.

Slow Down

quizzes and tests

Quiz and test corrections for “points”

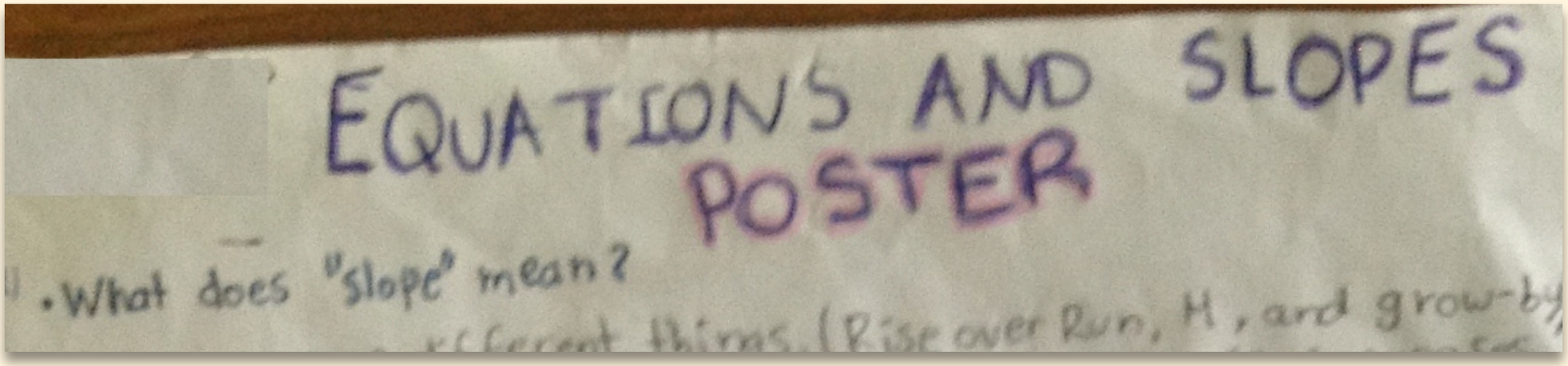
◊ You can get help — specify who helped you

◊ Full explanations in your own words

Reducing time pressure makes a **growth mindset** plausible!

Use cumulative tests and other assessments

Value at-home assessments



Students may impress you!

Reaching the full range

Make support available outside of class

Practical Strategies

Big-Picture Planning

Front-load difficult topics

(if they are important)

Put Some Time

between related units

Proportions … Dilation

Tangent … Sine, cosine

Separating related topics

◊ Proportions … Dilation
◊ Tangent … Sine and cosine
◊ Exponents … Scientific notation
◊ Linear functions … Systems of equations
◊ Exponentials … Logarithms
◊ Sequences … Series
◊ …

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Built-in review!

Two units at a time?

Circles	Quadrilaterals	Construction	
n^{th} Power Variation	Systems	Trigonometry	Review

Slows down each unit

Helps navigate both

Sends a message

These structural strategies help to extend exposure *without taking more time, purchasing materials, or requiring professional growth.*

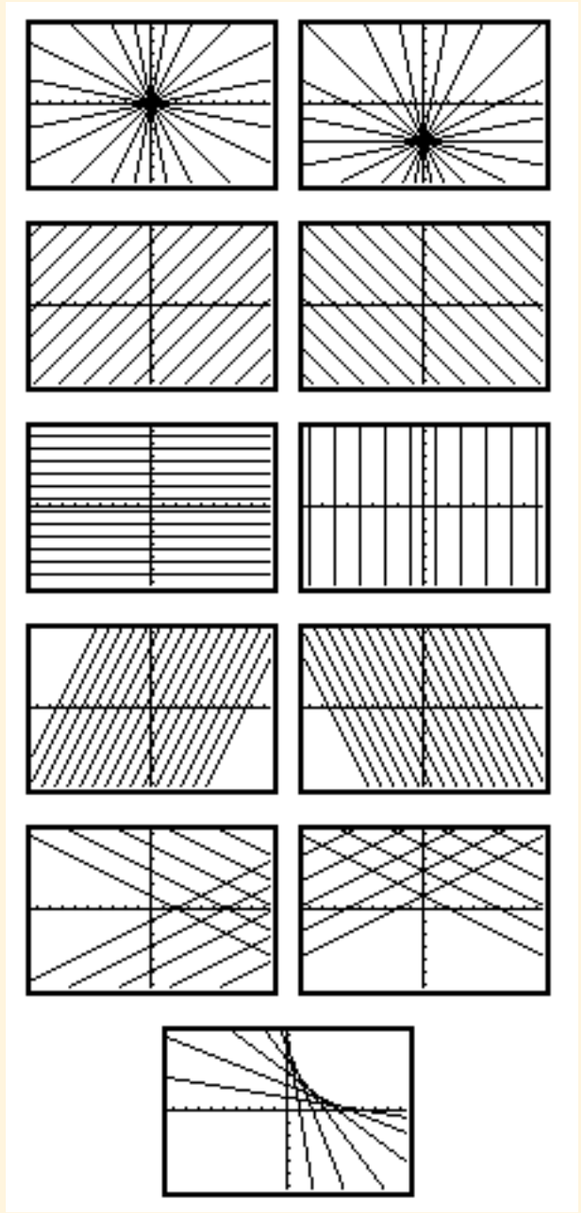
Longer-Term Strategies

Problem-Centered Curriculum

If necessary, find or adapt problems.

For example, instead of “graph this, graph that, what do you notice?”

Make These Designs



Group Work

◊ Random groups
  - new groups every 2 weeks

◊ Students (mostly) work independently
  - are expected to help each other

◊ If a group does not function well
  - intervene directly to get the behaviors you want

◊ If more than one group is stuck
  - stop them all for a class discussion

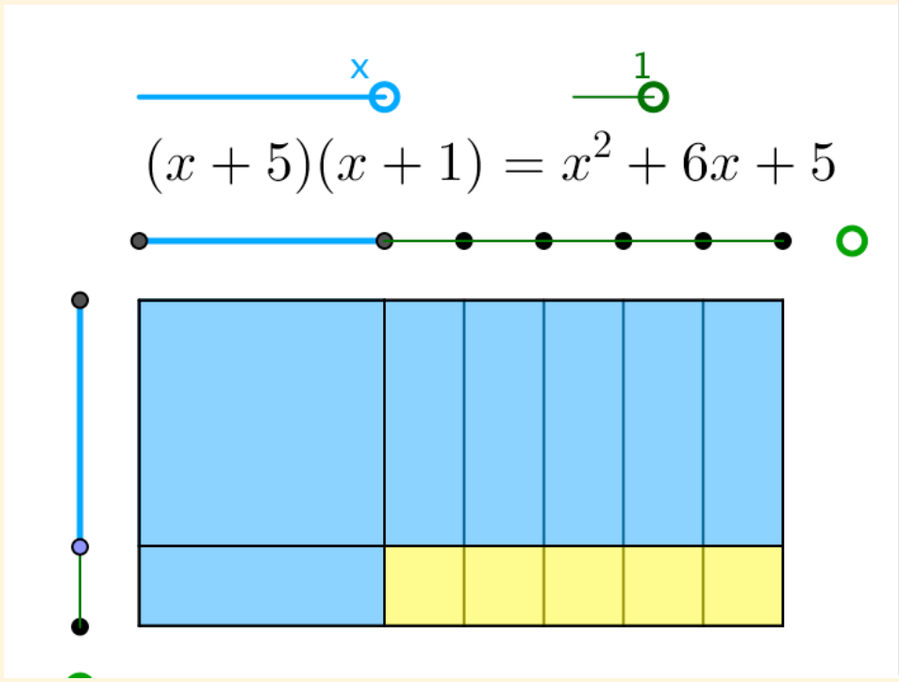
Whole-Class Discussion Techniques

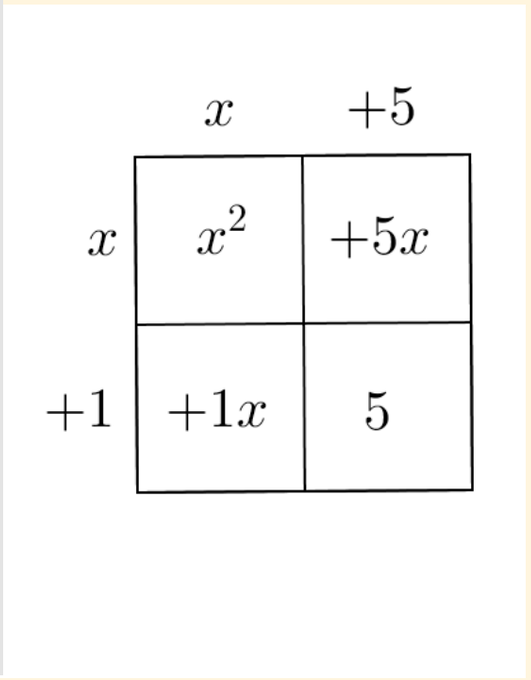
- **Time**: wait; count hands; tell your neighbor; rephrase; give a hint
- **Response mode**: on your fingers; in the air; on paper; all together
- **Agree/disagree**; votes
- **Etc.**: total silence; move around

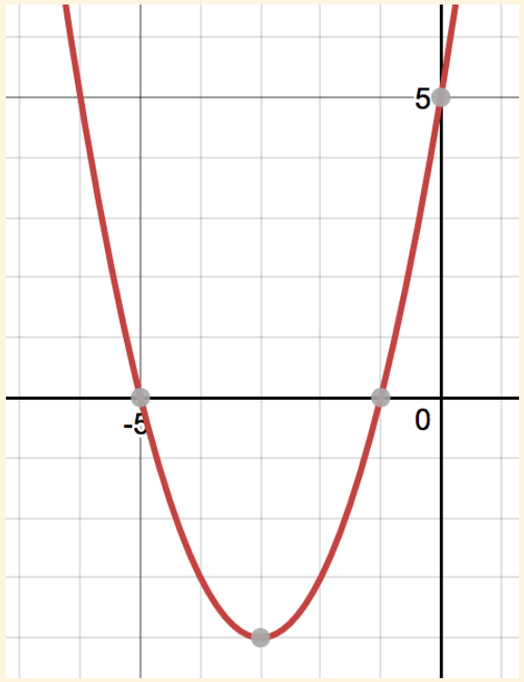
A tool-rich pedagogy

Slow Down

each unit





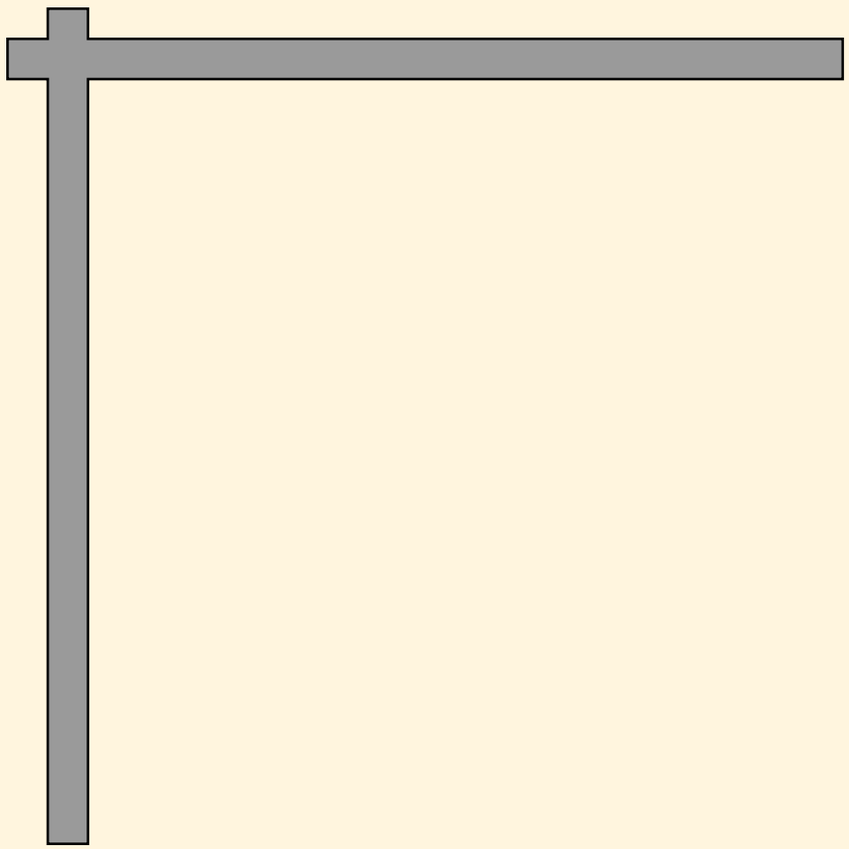


x	y_1
-5	0
-3	-4
-1	0
0	5

Teach important ideas many ways

Make a Rectangle

$$2x^2 + 4x$$



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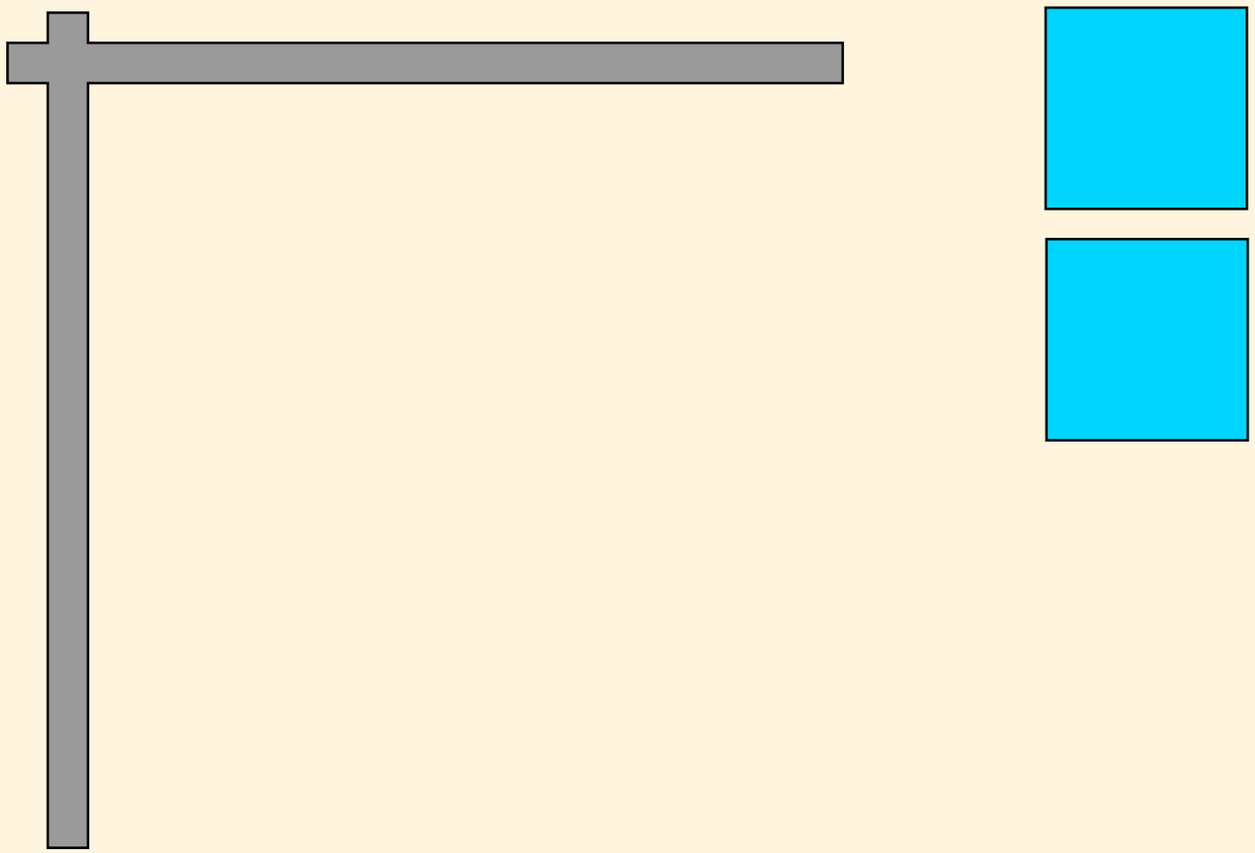

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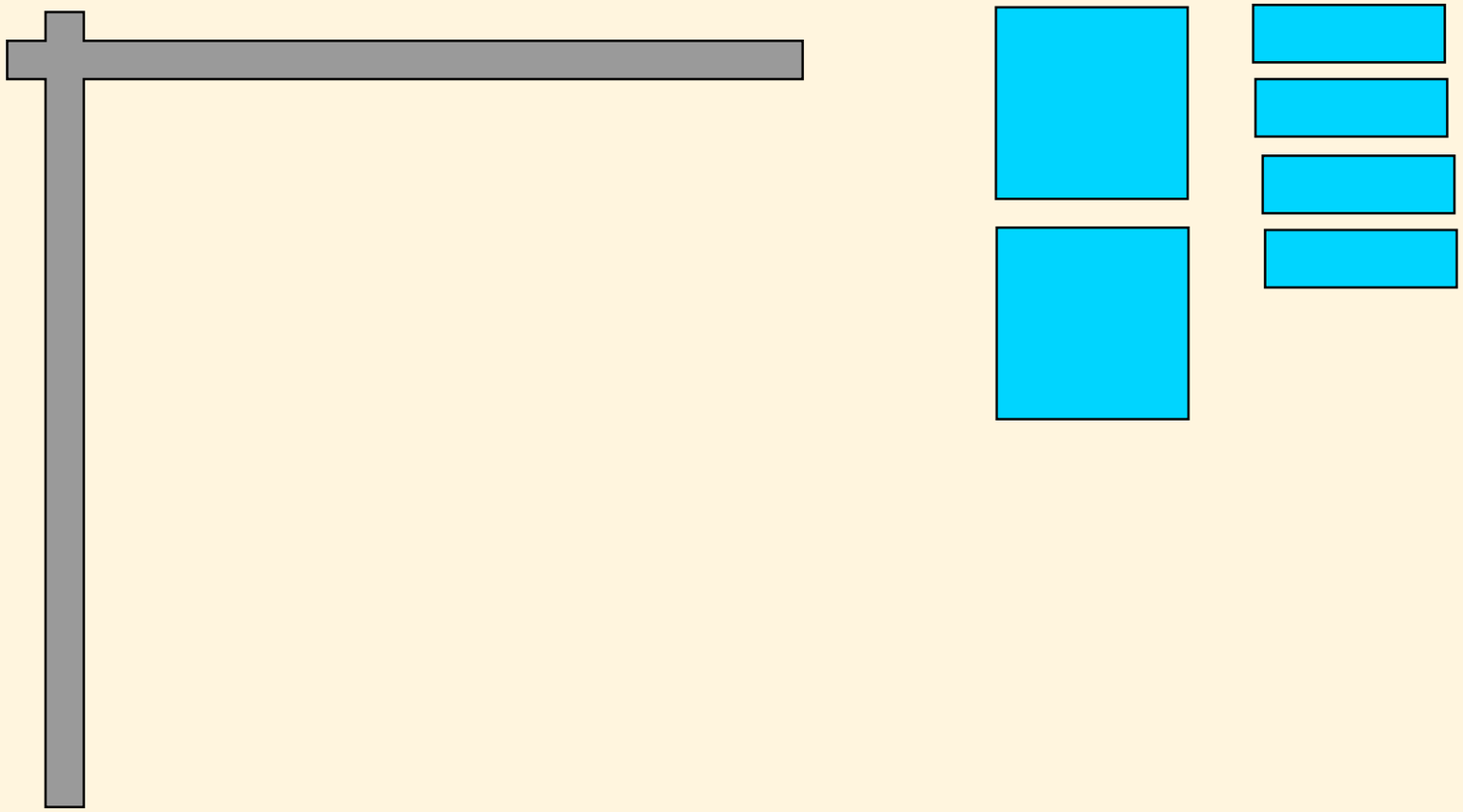

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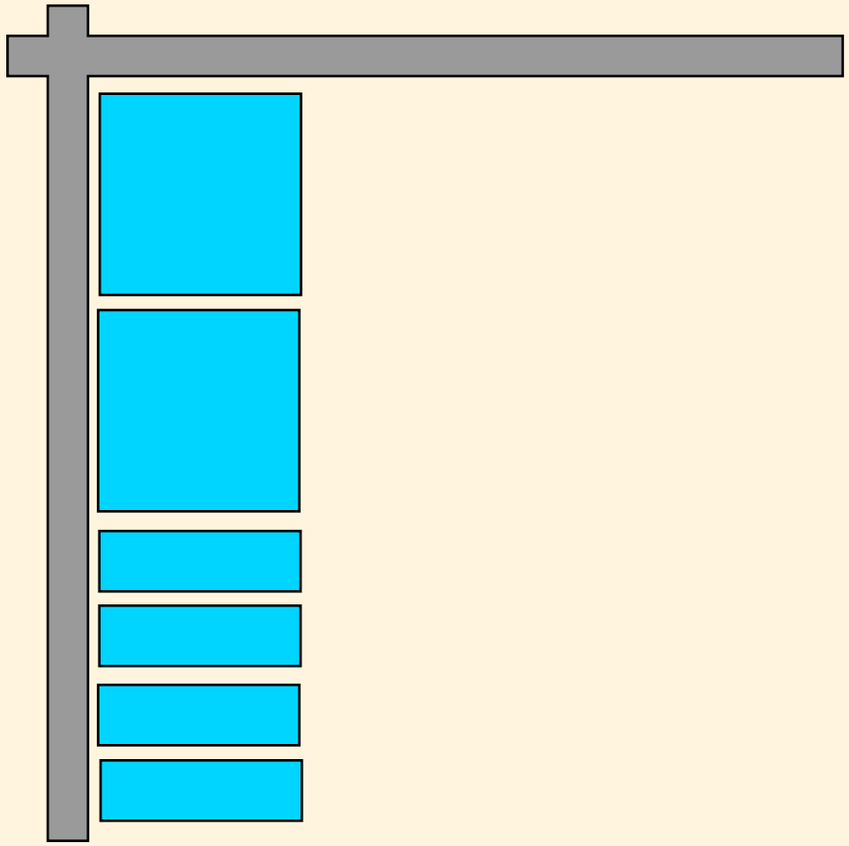

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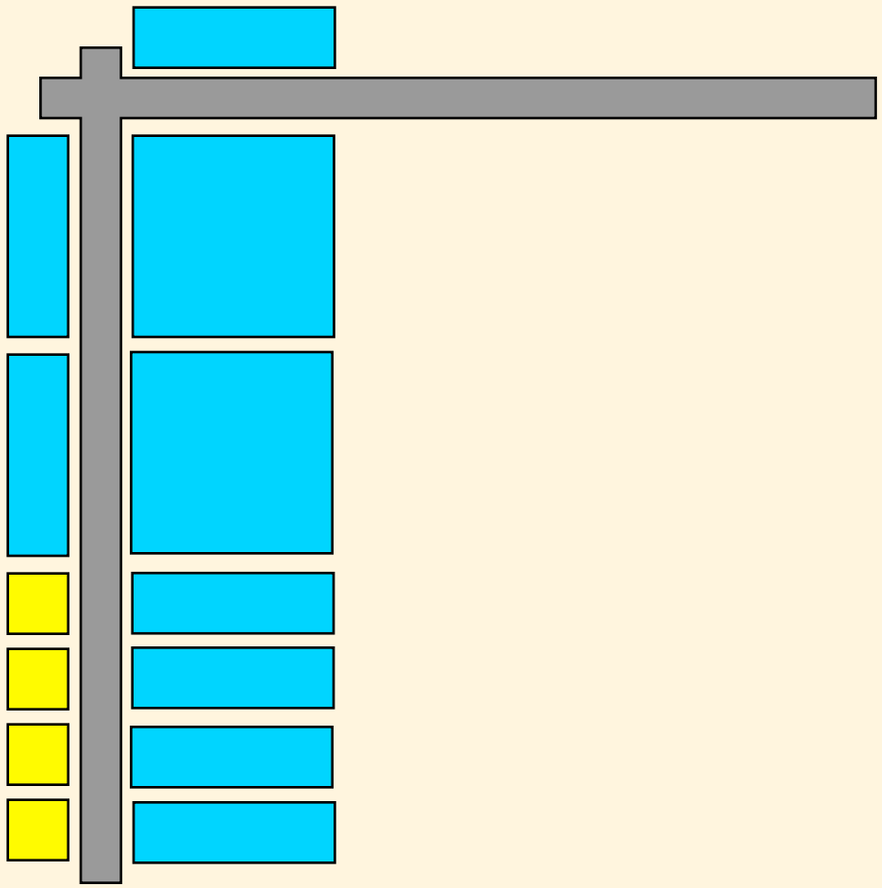

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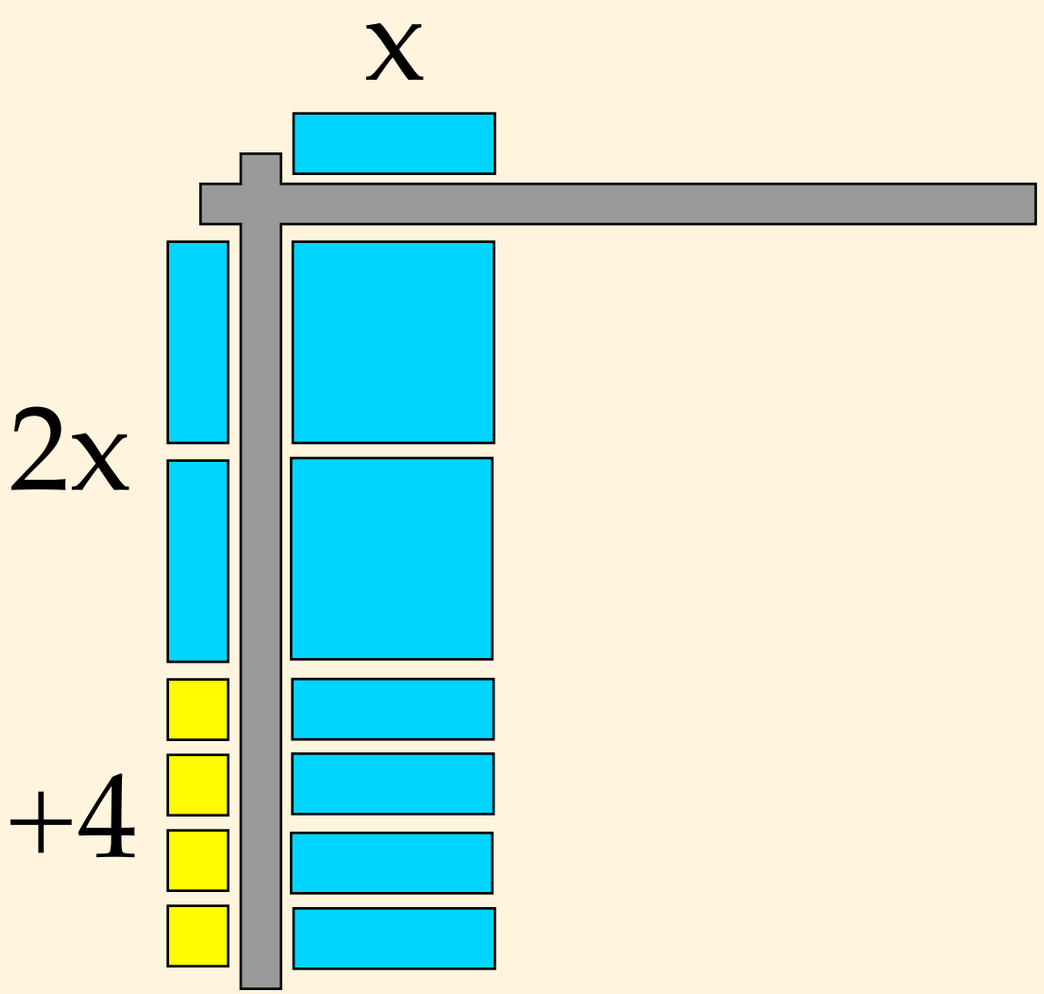

$2x^2 + 4x$

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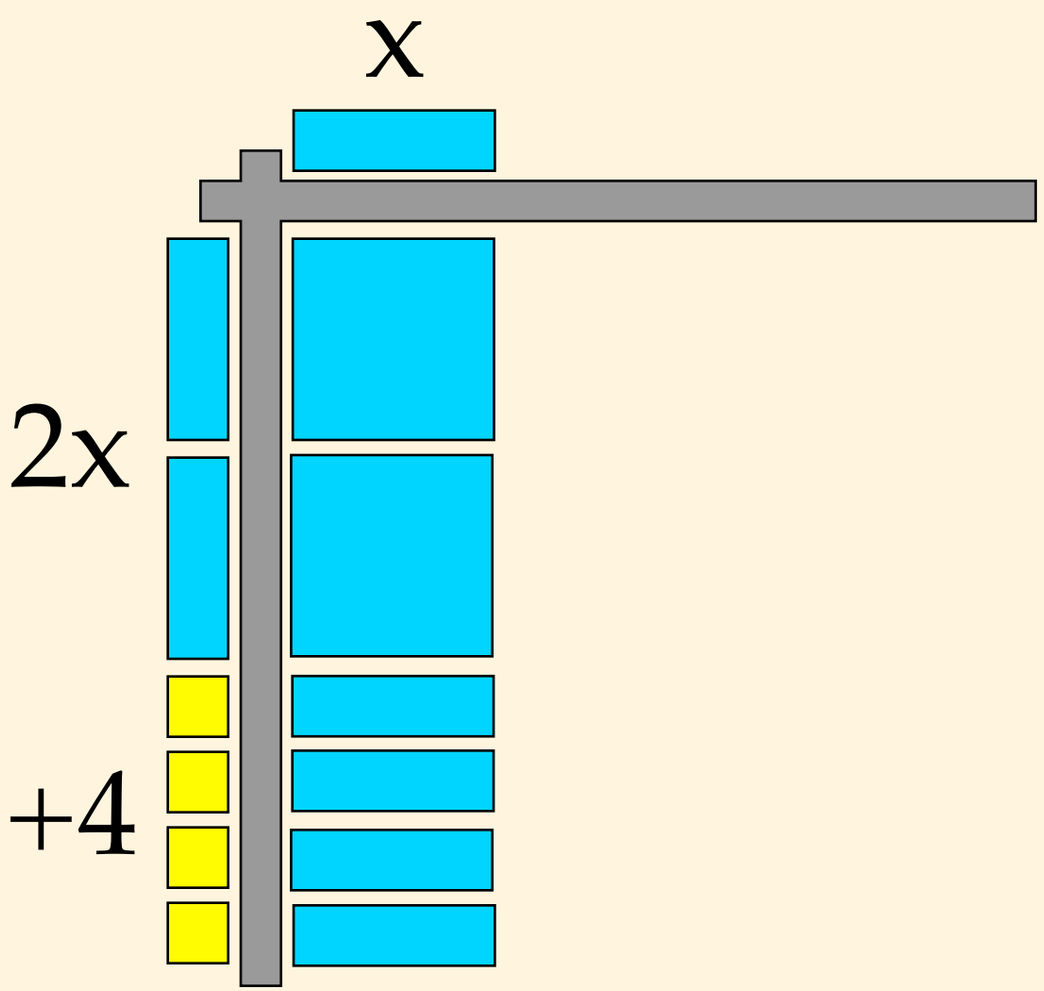
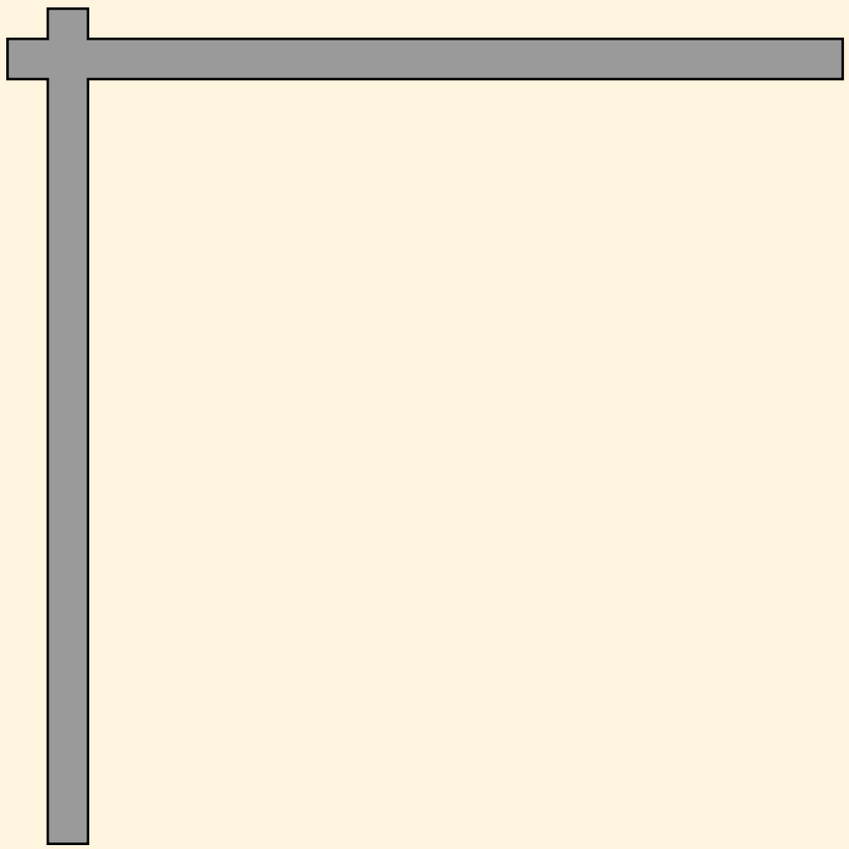


$$2x^2 + 4x = (2x + 4)\cdot x$$

Make a Rectangle

$$2x^2 + 4x$$



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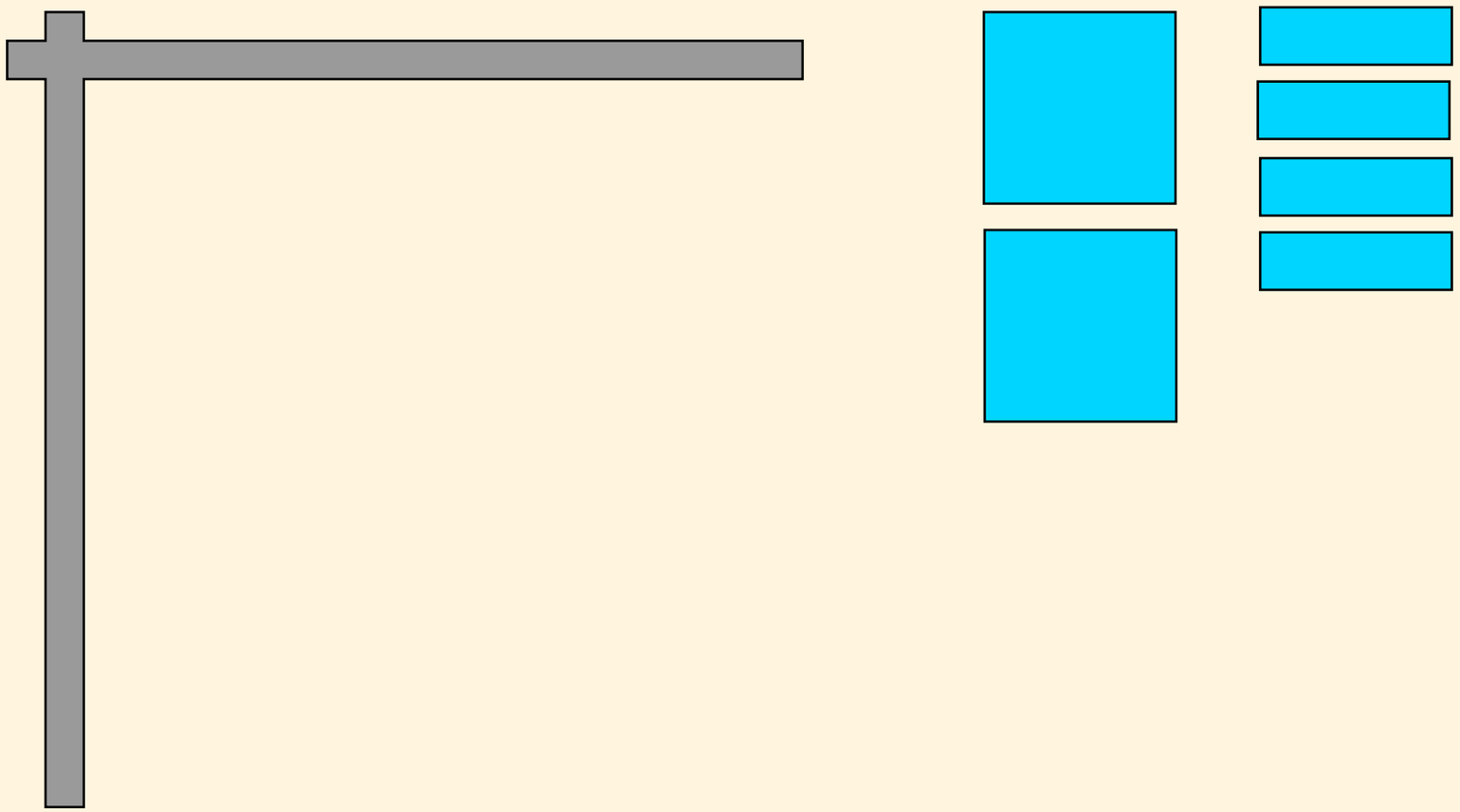

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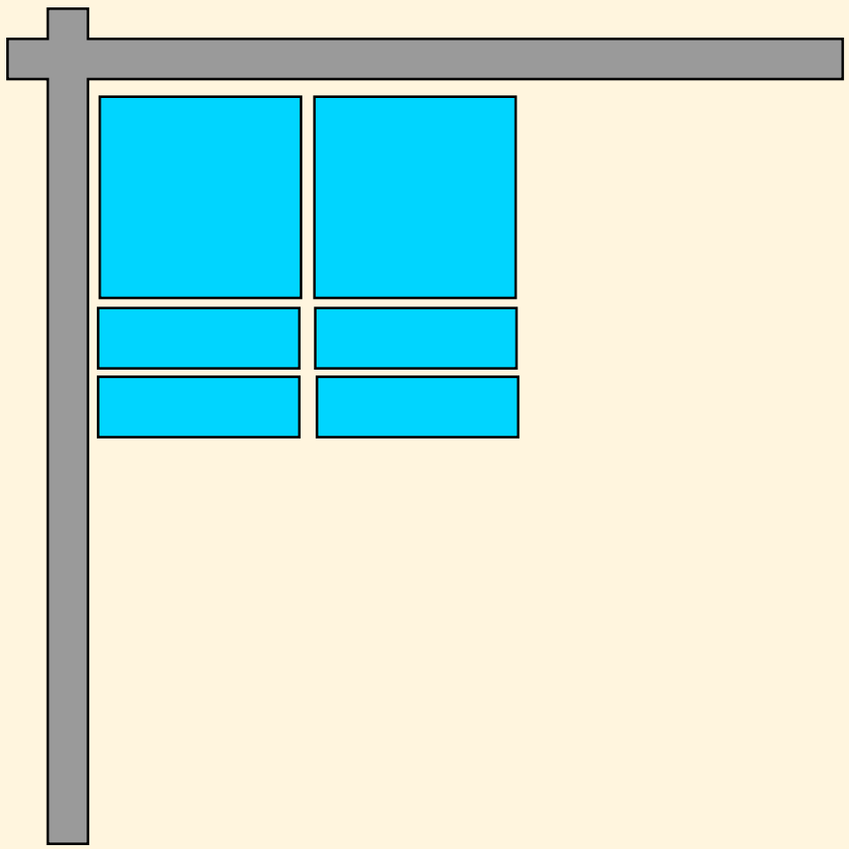

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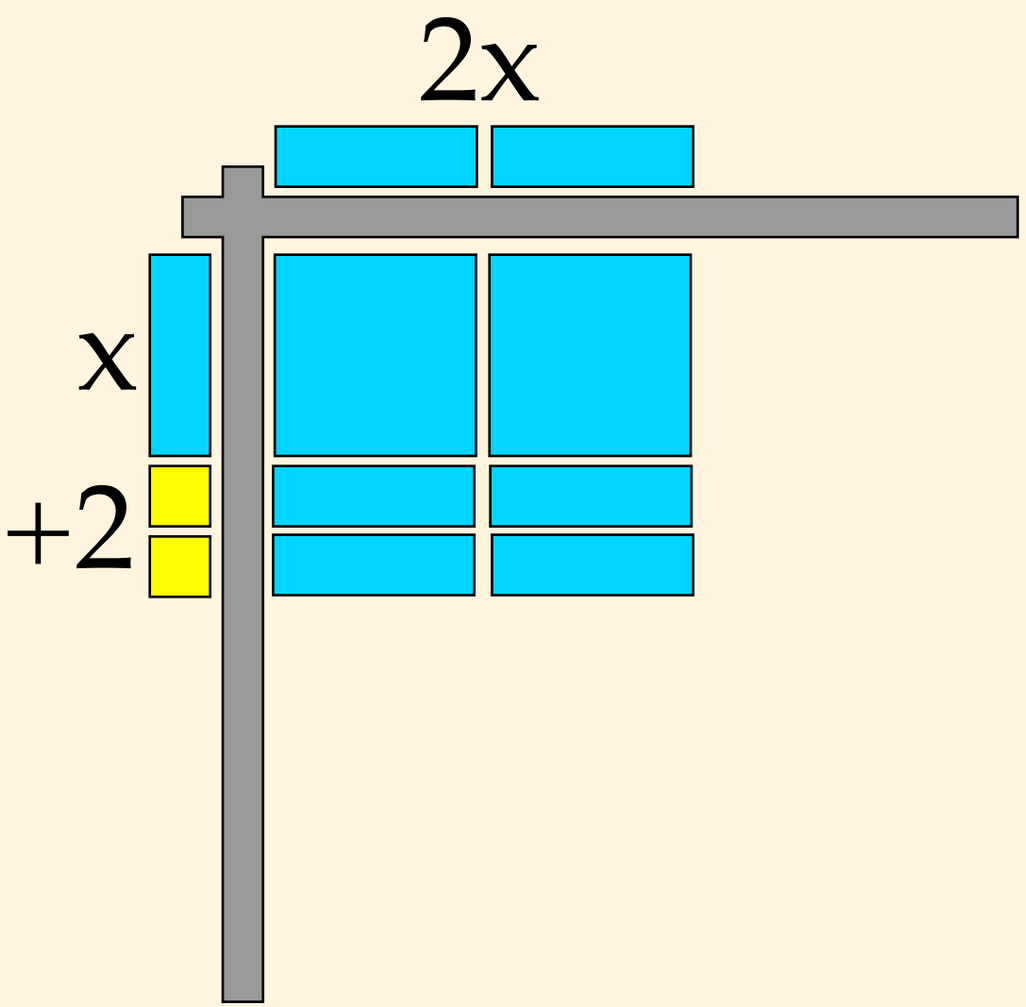
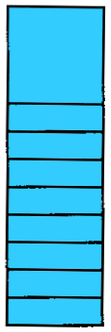
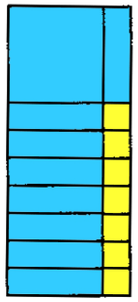


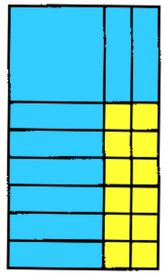
$$2x^2 + 4x = 2x \cdot (x+2)$$



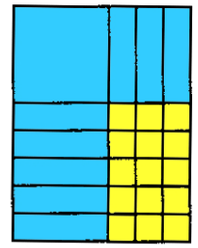
$(x + 8)x$
$= x^2 + 8x$



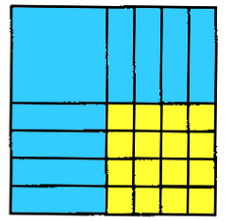
$(x + 7)(x + 1)$
$= x^2 + 8x + 7$



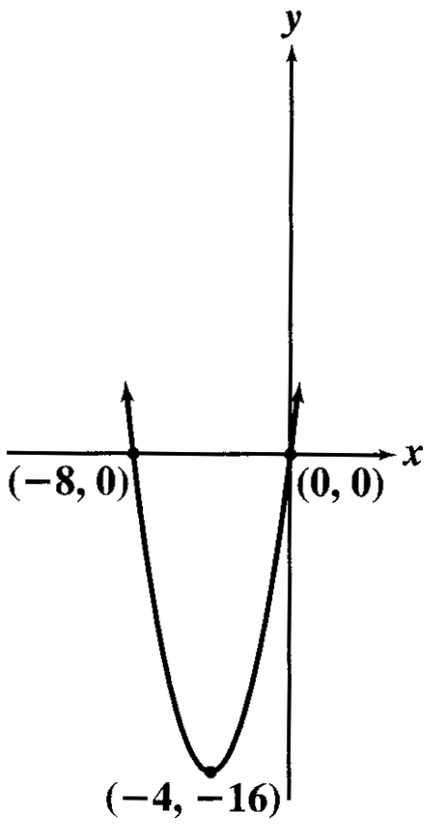
$(x + 6)(x + 2)$
$= x^2 + 8x + 12$



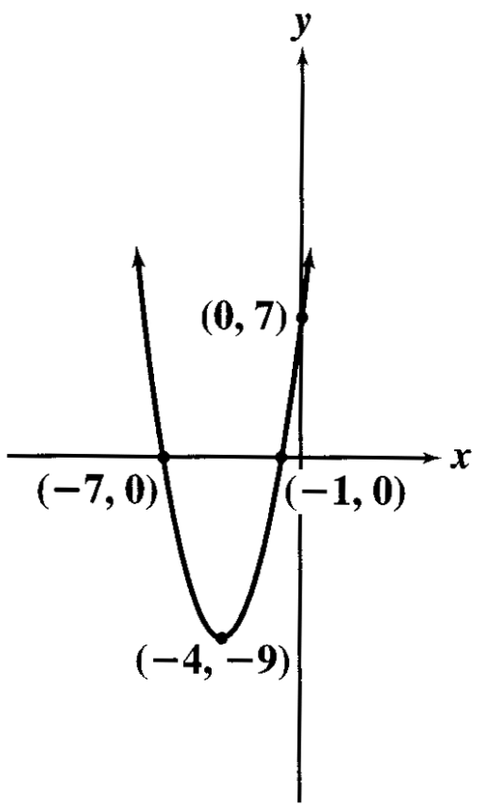
$(x + 5)(x + 3)$
$= x^2 + 8x + 15$



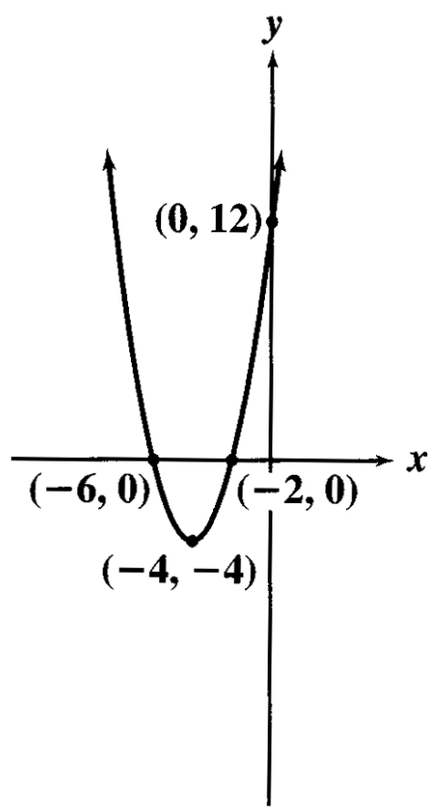
$(x + 4)^2$
$= x^2 + 8x + 16$



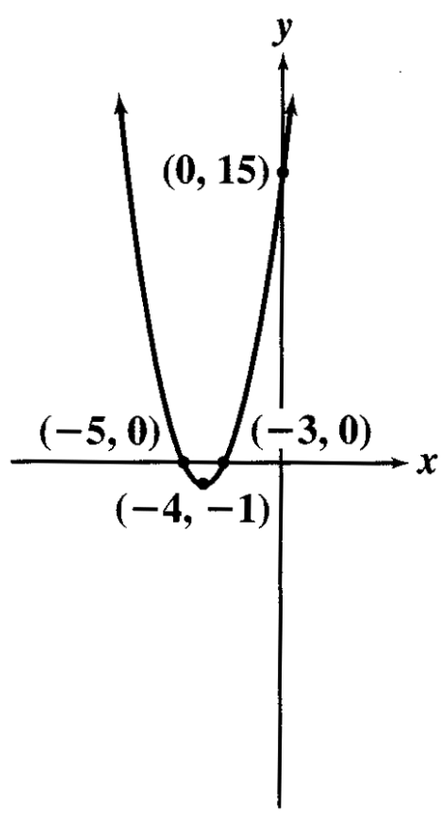
$y = x^2 + 8x$



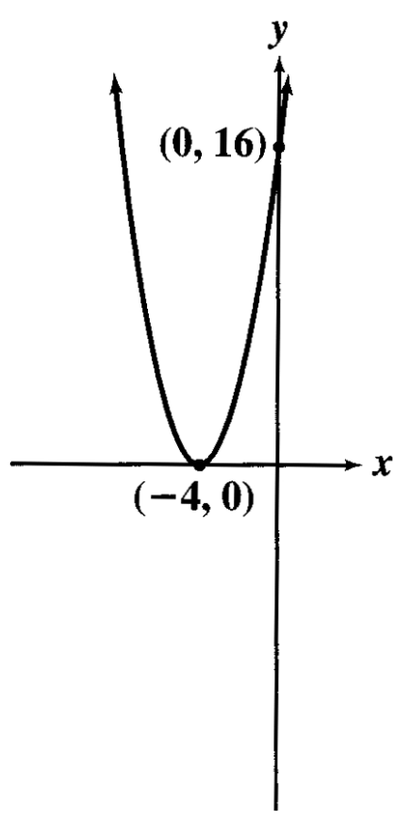
$y = x^2 + 8x + 7$



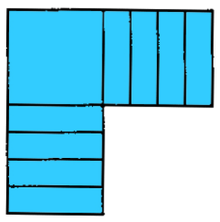
$y = x^2 + 8x + 12$

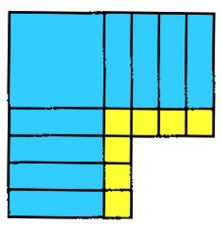


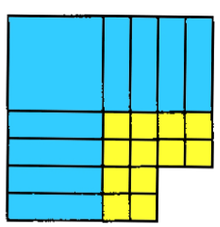
$y = x^2 + 8x + 15$

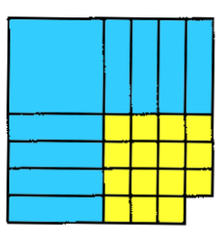


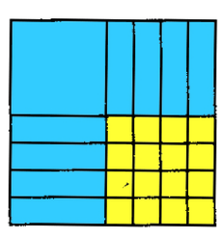
$y = x^2 + 8x + 16$











From *Algebra Lab Gear: Algebra 1*, by Henri Picciotto (© Didax, 2016)

Find many geoboard squares
of different sizes (and their areas).

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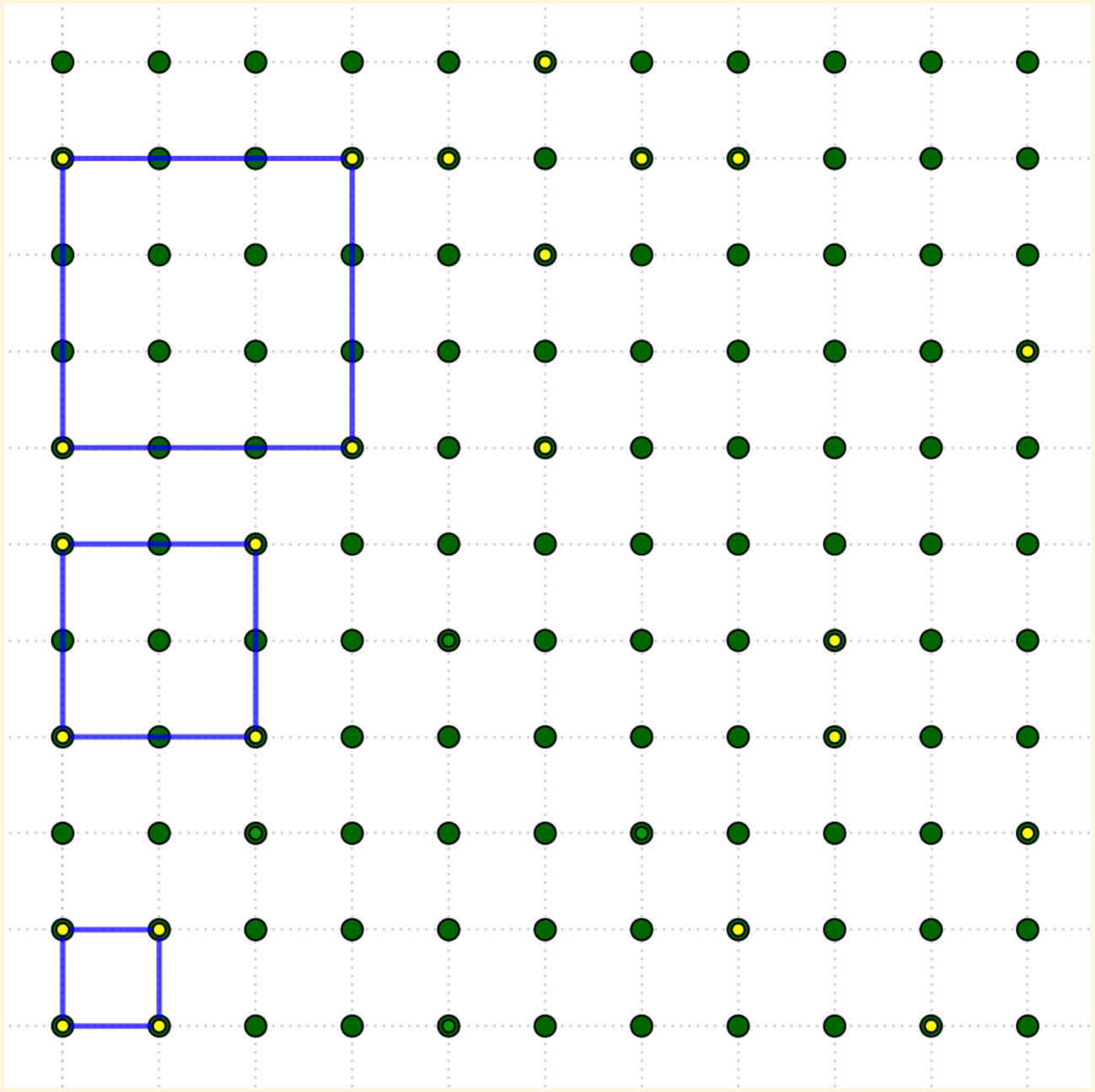
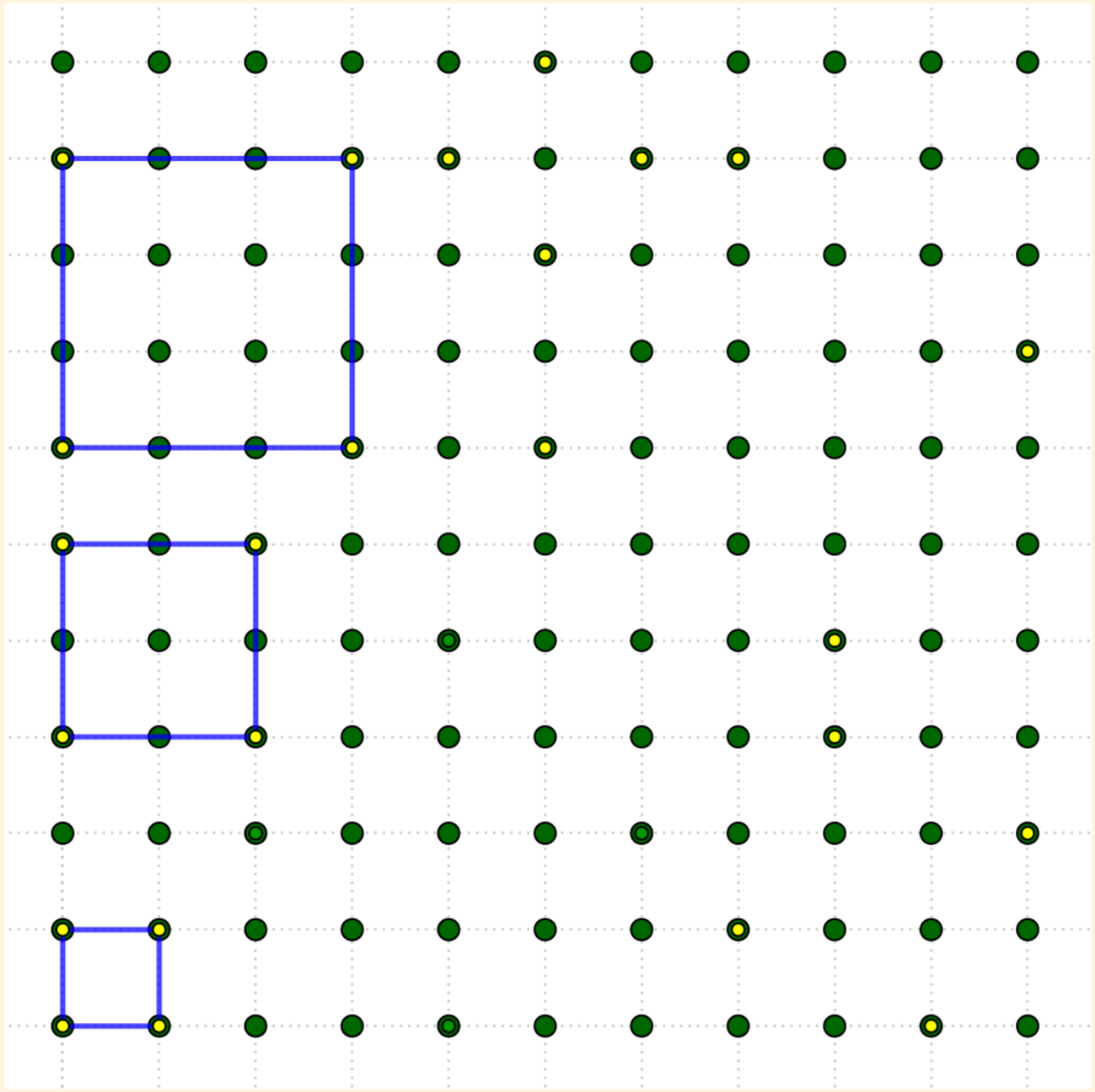


Find many geoboard squares of different sizes (and their areas).

Hint: there are more than 10



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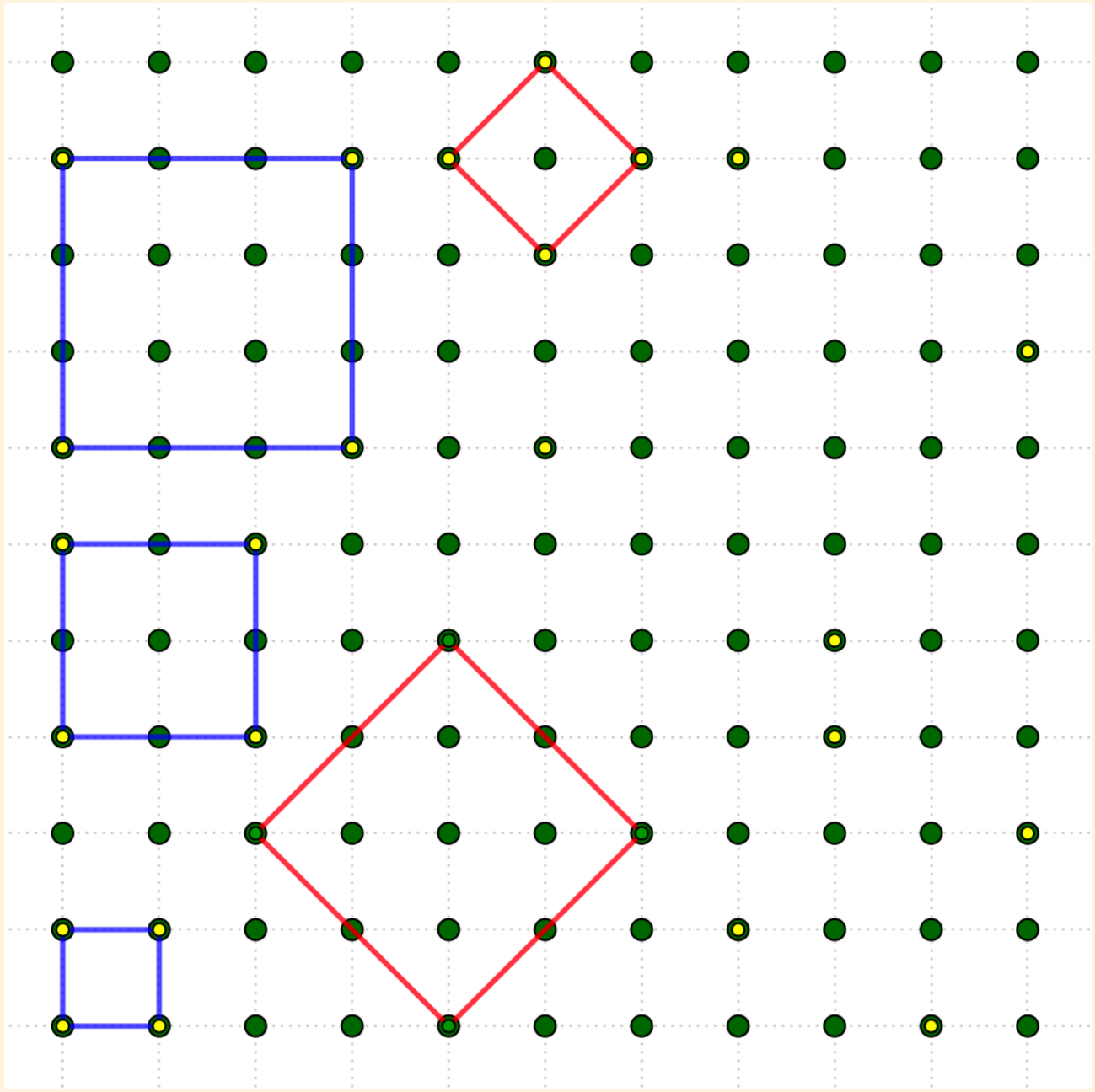

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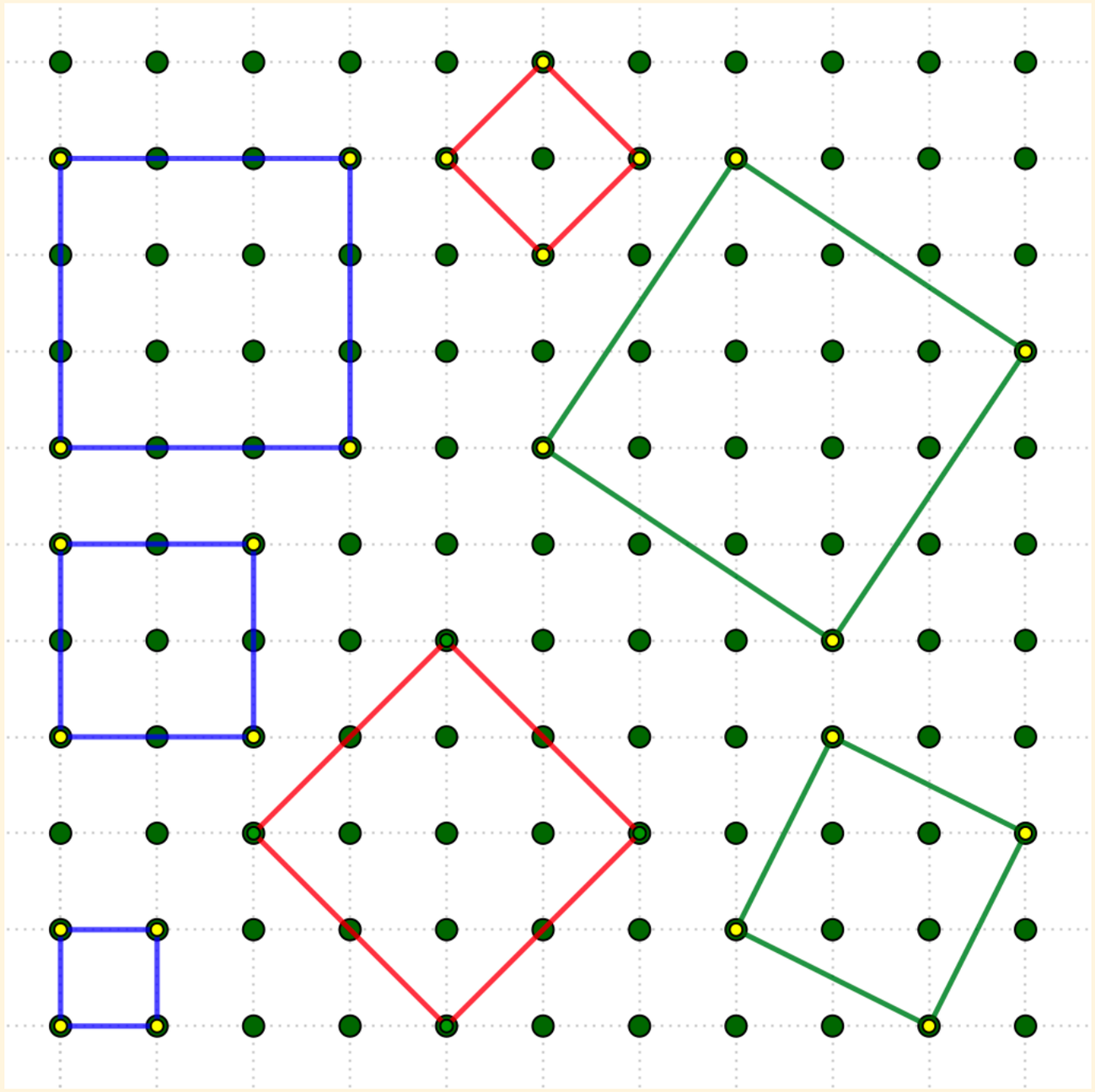
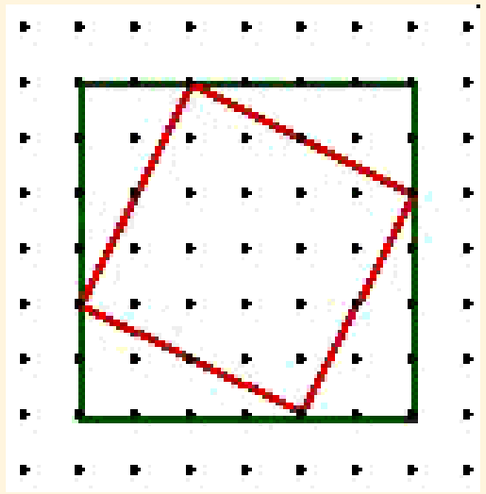
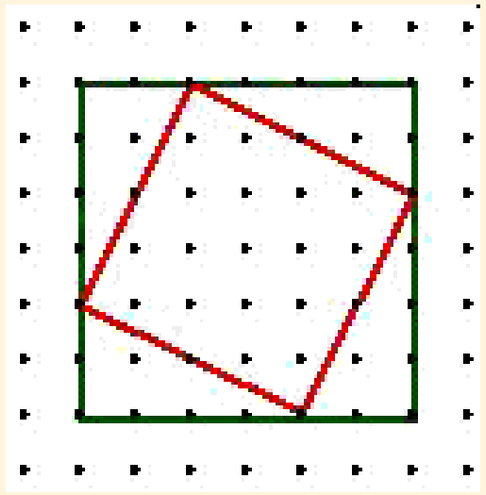




What is the area of the inner square?



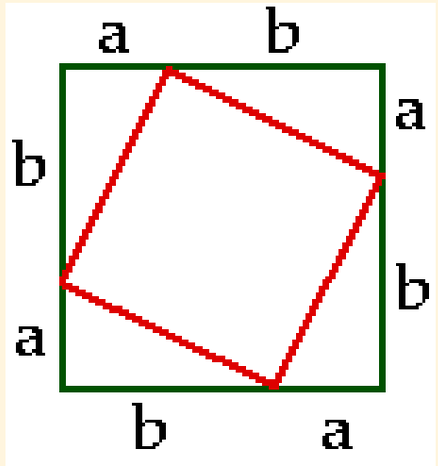
What is the area of the inner square?

outer square: 6^2

each triangle: $\frac{2 \cdot 4}{2} = 4$

inner square: $36 - 4 \cdot 4 = 20$

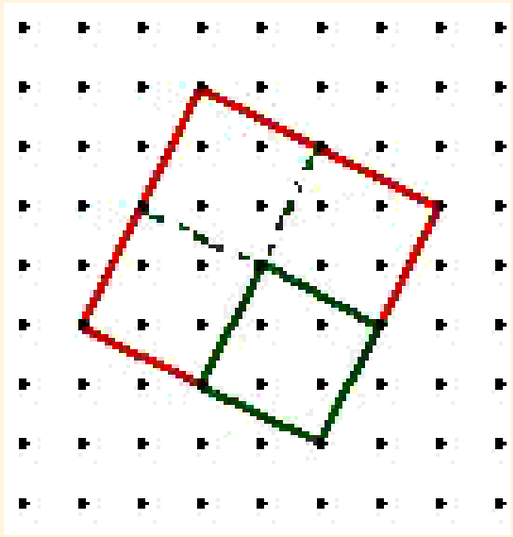
Generalize



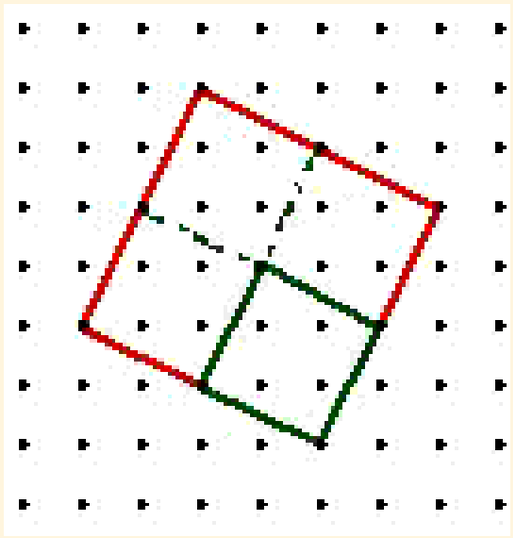
outer square: $(a + b)^2$

each triangle: $\frac{a \cdot b}{2}$

inner square: $(a + b)^2 - 2ab = a^2 + b^2$

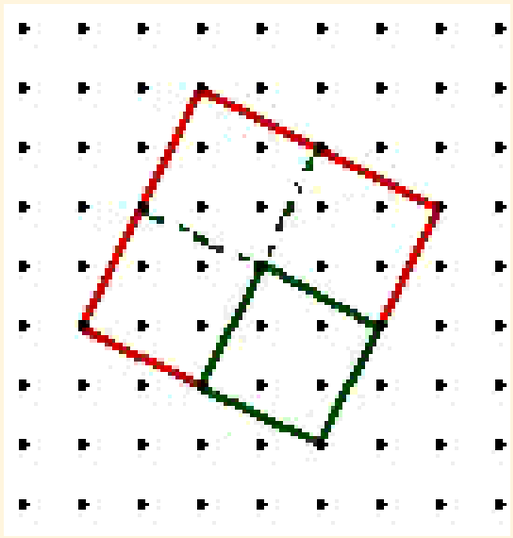


Area of the red square: 20



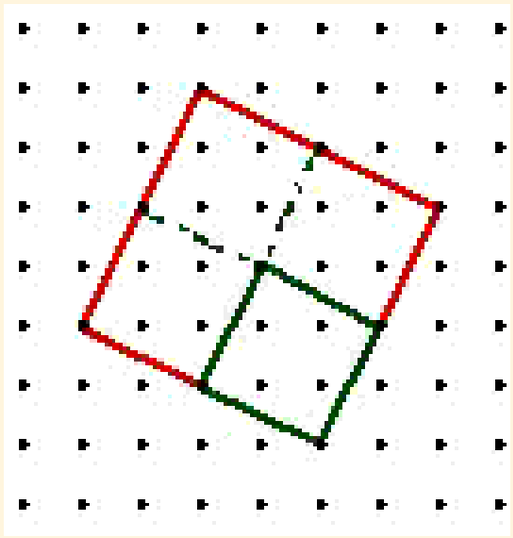
Area of the red square: 20

Area of the green square:



Area of the red square: 20

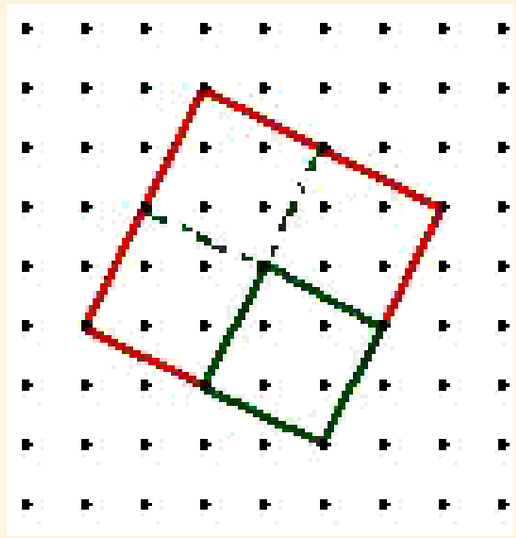
Area of the green square: 5



Area of the red square: 20

Area of the green square: 5

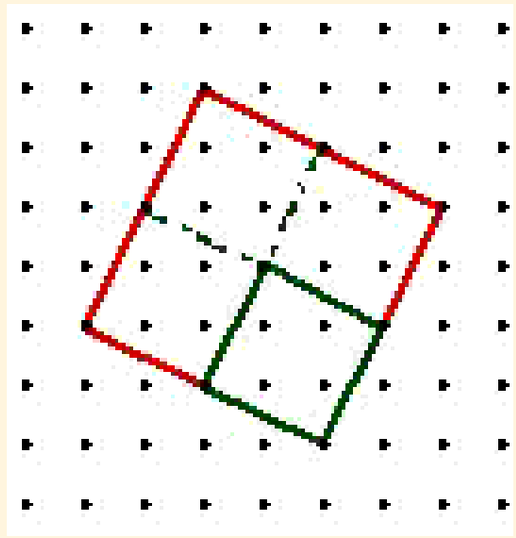
Side of the red square:



Area of the red square: 20

Area of the green square: 5

Side of the red square: $\sqrt{20}$

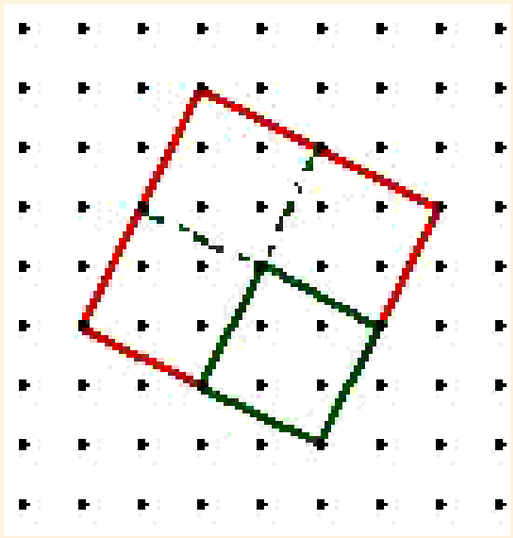


Area of the red square: 20

Area of the green square: 5

Side of the red square: $\sqrt{20}$

Side of the green square:

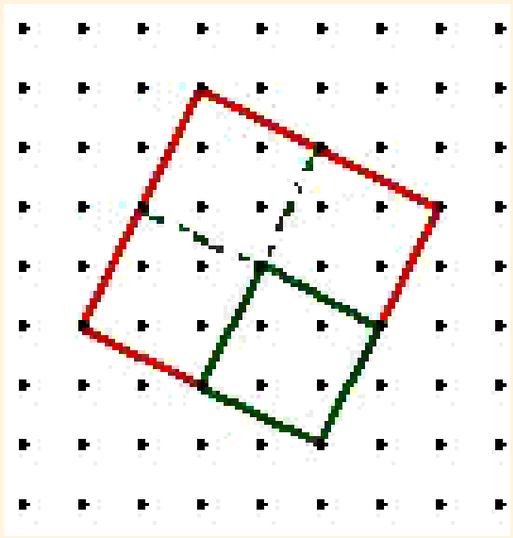


Area of the red square: 20

Area of the green square: 5

Side of the red square: $\sqrt{20}$

Side of the green square: $\sqrt{5}$



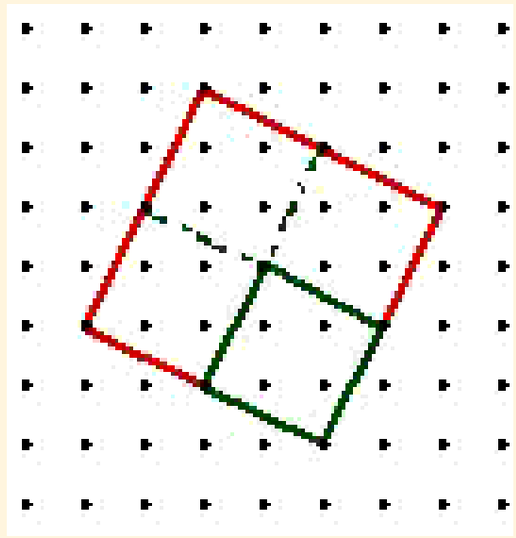
Area of the red square: 20

Area of the green square: 5

Side of the red square: $\sqrt{20}$

Side of the green square: $\sqrt{5}$

Conclusion:



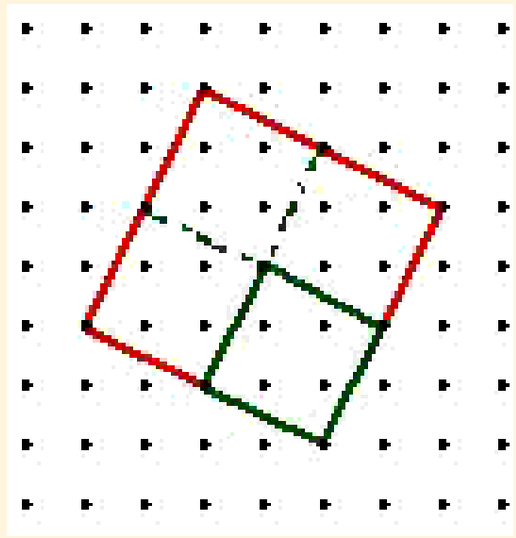
Area of the red square: 20

Area of the green square: 5

Side of the red square: $\sqrt{20}$

Side of the green square: $\sqrt{5}$

Conclusion: $\sqrt{20} = 2\sqrt{5}$



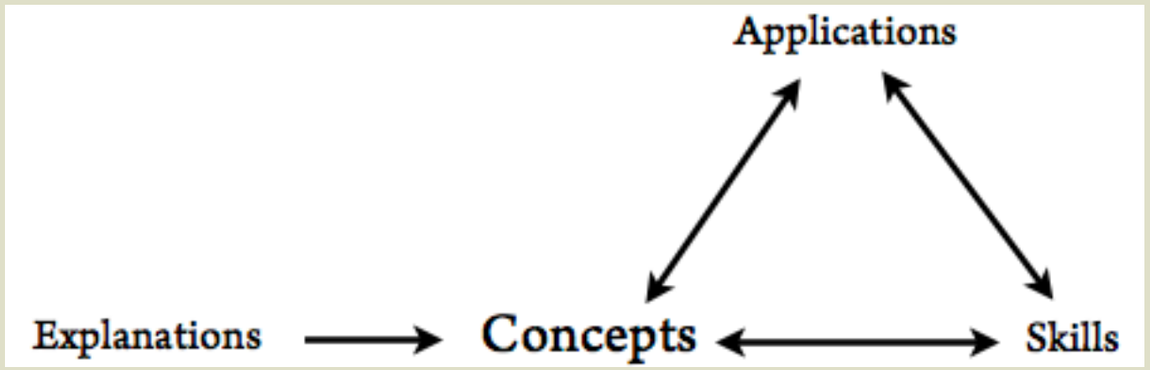
Area of the red square: 20

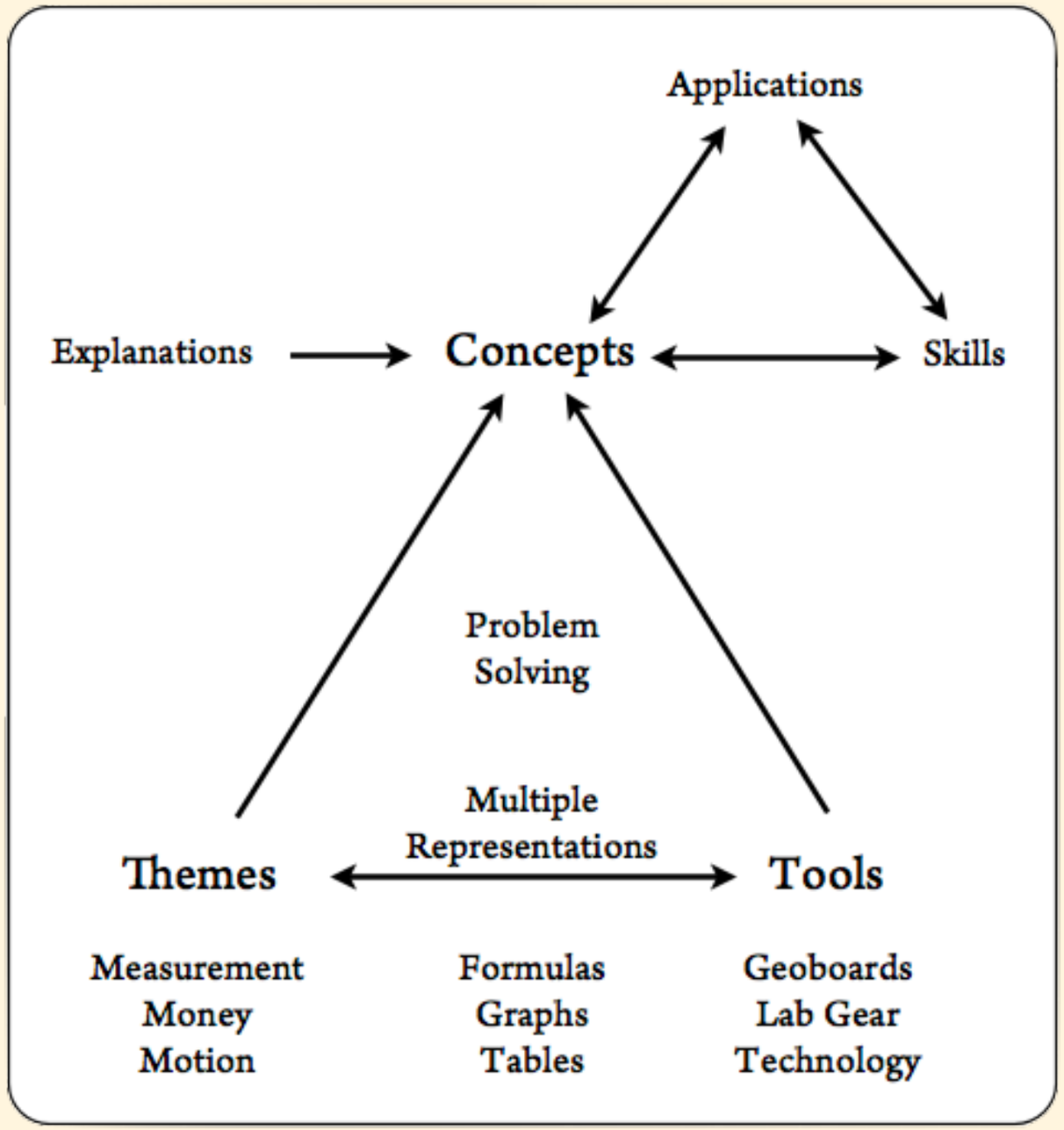
Area of the green square: 5

Side of the red square: $\sqrt{20}$

Side of the green square: $\sqrt{5}$

Conclusion: $\sqrt{20} = 2\sqrt{5}$!


Applications
Explanations
Concepts
Skills


Applications
Explanations
Concepts
Skills
Problem
Solving
Multiple
Representations
Themes
Tools
Measurement
Money
Motion
Formulas
Graphs
Tables
Geoboards
Lab Gear
Technology

Professional Growth

Know Thyself

Know Thyself

forward motion
review

forward motion
review

forward motion
review

More Examples

over-prepared vs. winging it
routine vs. variety
enjoyment vs. learning
correct vs. incorrect answers
intrinsic vs. extrinsic motivation

Find your profile!

Find your profile!

Lose your profile!

Lose your profile!

forward motion
review

forward motion
review

forward motion
review

Embrace contraries

Be eclectic and flexible

Learn to navigate along the axes

Conference and Speaker Evaluation

https://tinyurl.com/CMCNEVAL

There Is No One Way!

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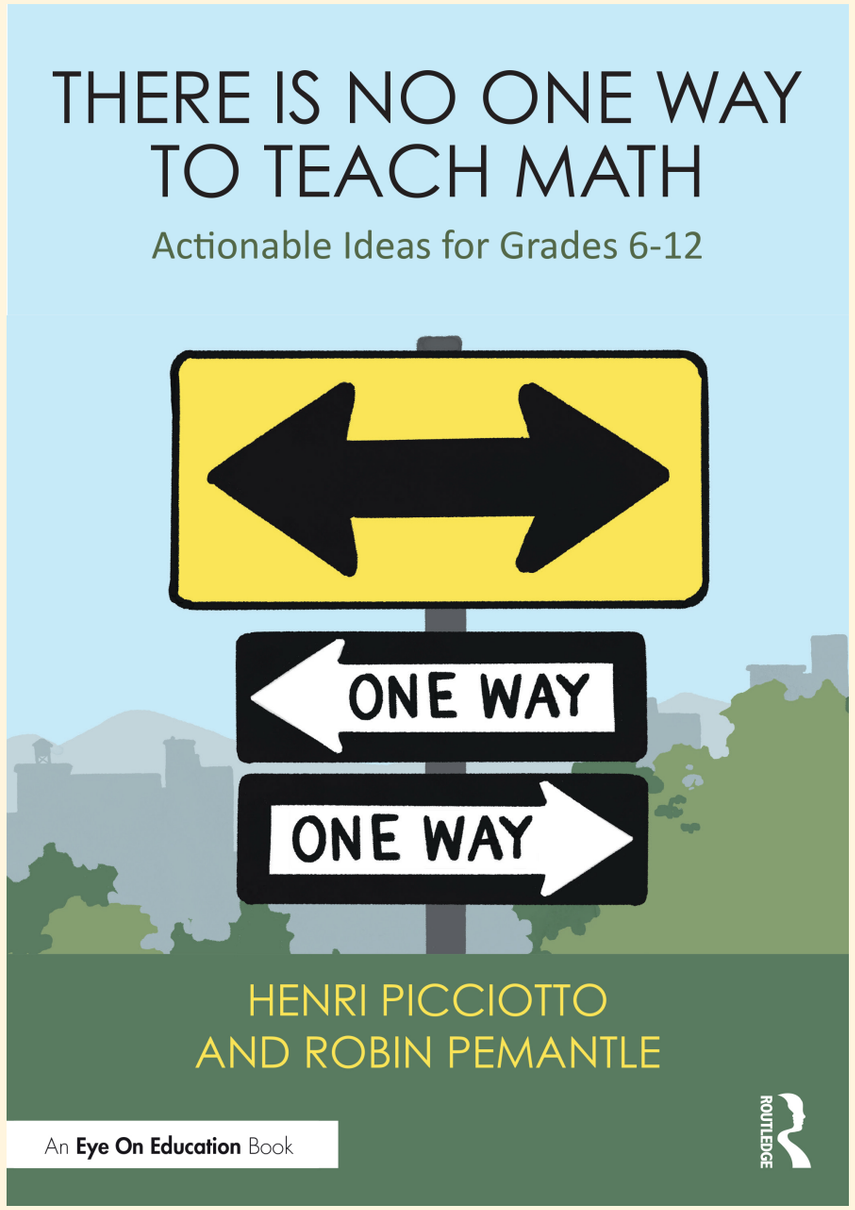

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