Nothing Works! The Art of Teaching Mathematics

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The Art of Teaching

• Find your profile!
• Example
  • Pasted Graphic
• More examples
  • staying within math vs. making connections
  • over-prepared vs. winging it
  • routine vs. variety
  • enjoyment vs. learning
  • correct vs. incorrect answers
  • intrinsic vs. extrinsic motivation
• Lose your profile!
  • Learn to navigate along these axes

Nothing Works

• Heterogeneous Classes
  • All classes are heterogeneous
• Pedagogy
  • Alliance with the strongest students
    • Support for the weakest
  • The elevator strategy
    • Stop on all floors
      • Something too difficult
      • Something too easy
      • Something "just right"
  • Pacing
• Constant forward motion
  • Eternal review
• Curriculum
  • "Vertical" activities which provide both access and challenge
  • "No threshold, no ceiling"
    • Example: what perimeters are possible for a given graph paper shape?
      • Many more examples on my Web site
• Tools
  • Manipulative and technogical tools
    • Calculator: TI-89
    • Manipulatives: cubes, geoboards, Lab Gear, pattern blocks, ten-sided dice, ...
    • Software: Cabri, Fathom, ...
• Multiple representations of concepts
  • Numeric, symbolic, graphical, geometric, applied, ...
    • To provide an entry point to more students
    • To preview or review concepts
    • To extend exposure
      • To deepen understanding
    • To increase variety
      • To promote engagement
• However...
  • tools are not magic
• Group Work
  • Random groups
    • new groups every 2 weeks
  • Students mostly work independently
    • are expected to help each other
  • If a group does not function well
    • I intervene directly to get the behaviors I want
• If more than one group is stuck
  • I stop them all for a class discussion
  • guide on the side vs. sage on the stage

• Discovery

• Discovery vs. Direct Instruction
  • A false choice: neither works well without the other
    • After exploration, "institutionalization"
      • Make key concepts explicit
        • students may not get there on their own
      • Clarify what is important and worth remembering
        • and thus worth writing down
    • Make connections
      • with other representations
      • with previous knowledge
      • "Nothing transfers"

• Verbalizing
  • Putting things in words is crucial to understanding
    • I encourage talking
    • I require writing

• Class Discussion
  • True Discussion vs. Interactive Lecture
    • Open-ended questions
  • Creating a safe environment
    • No putdowns
    • I praise participation and risk-taking
      • rather than correct answers
    • "Tell your neighbor..."
    • "Can you restate what X said?"
  • Handling wrong answers
    • write down many answers, then discuss
      • poker face vs. telling
      • "Choose someone to help you"
• Making 'mistakes' myself

• Feedback from all
  - votes
  - gestures
  - writing

• Variety
  - Fanfare vs. total silence
  - New problems, not same as on paper
  - Move around the room

• Homework
  - I keep it reasonable
    - most learning happens at school
  - I keep it separate from class work
    - less rushing, more cooperation
  - "Lagging"
    - Pasted Graphic
      - (constant forward motion, eternal review)

• Assessment
  - Purpose
    - To improve teaching
      - Diagnose student understanding and skills
        - Figure out next steps and generally fine-tune the course
    - To improve learning
      - Let students know where they are
        - Provide learning opportunities
  - Also...
    - Prepare students for future assessments (!)
      - Rank students / assign grades
        - Justify the grades

• Variations on the quiz/test routine
  - Participation quiz
    - Occasional take-home assignments
      - Test corrections
  - I keep it manageable
I don't write extensive comments on tests

because...

- When correcting work, I'm working for one student
  - When planning, I'm working for the whole class
    - A true passion for math and learning is not triggered by assessment or grades

Sequencing topics

Overall

- The weight of tradition
  - quadratic formula in Algebra 1,
  - exponential functions in Algebra 2
- Topics can and should move if they are
  - too early (with respect to student's development)
  - too late (more accessible thanks to new approaches)

Within a course

- do important and/or difficult topics early
  - Example: inscribed angles near beginning of Geometry

- separate related topics
  - tangent / sine and cosine
  - exponentials / logarithms
  - sequences / series

Navigating a topic

- concrete to abstract, and back
  - positive whole numbers to rational numbers
- numbers to variables
  - discrete to continuous
    - Example: the Pythagorean theorem on the geoboard
- concepts to vocabulary and notation, and back
  - Example: trig ratios on the ten-centimeter circle
- difficult to easy, and back

Teaching for Understanding

Skills vs. concepts
Another false choice!

In part because of technology,

- speed and accuracy are no longer legitimate priorities for math education
  - understanding is more important than ever

A student who understands a concept can

- explain it
- reverse processes associated with it
  - distribute $\leftrightarrow$ factor
- flexibly use alternative approaches
  - e.g. to equation solving
- successfully handle non-rote assessments
- navigate between multiple representations

Understanding...

- is difficult to encapsulate in a checklist
  - cannot be easily conferred by explanations
    - is difficult to assess
    - is not always valued by students and parents
      - is the most important part of our job

Nothing works...

- ...for every student, every class, every teacher, every day

I am skeptical of claims that some particular approach is the answer

- whether 'traditional' or 'reform'

I don't throw away or rule out any technique

I try to constantly broaden my repertoire

- I am eclectic

**The Art of Learning**

- Teaching / Learning

- In the end, there is no teaching...
  - ...only learning
  - Thus the goal: self-motivated students
· Our own learning
  · about math
    · about learning
      · is what makes the job interesting in the long haul
  · There is no one way