# Activity: Symmetric Polygons

Name and sketch the polygon that could be defined by the symmetry properties listed below:

1. A triangle with at least one line of symmetry.

2. A triangle with at least two lines of symmetry.

3. A quadrilateral with 180° rotational symmetry.

4. A quadrilateral with at least one line of symmetry that passes through vertices.

5. A quadrilateral with at least one line of symmetry that does not pass through vertices.

6. A quadrilateral with two lines of symmetry that pass through vertices.

7. A quadrilateral with two lines of symmetry that do not pass through vertices.

8. A quadrilateral with four lines of symmetry.

9. A polygon with $n$ lines of symmetry. Two cases:
   a. Each line of symmetry passes through either two vertices or through zero vertices.

   b. Each line of symmetry passes through exactly one vertex.

# Symmetric Polygons: Teacher Notes

**CONCEPTS:** Symmetry, special polygons

**SKILLS:** Recognizing special polygons and their symmetries, and using the corresponding vocabulary.

**MATH CONTENT STANDARDS:** G-CO.3, preparation for G-CO.10 and G-CO.11

**MATH PRACTICE STANDARDS:** 2, 3, 6, 7

**GRADES:** 8-10

**PREREQUISITES:** Students should know the names and the traditional definitions of the symmetric polygons (isosceles and equilateral triangles, the special quadrilaterals, and regular *n*-gons). If necessary, you should start with a review of these.

## NOTES

This activity could be a stand-alone activity, to help develop students' understanding of symmetry and deepen their familiarity with the special polygons. Or it could be used as a foundational activity in a unit about special polygons. In such a unit, the properties of special polygons can be deduced from their symmetry definitions. For example, "The diagonals of a parallelogram bisect each other." Conversely, certain properties can be shown to be sufficient to prove a given polygon is special. For example, "If the diagonals of a quadrilateral bisect each other, the quadrilateral must be a parallelogram." In a transformations-based geometry course, this would not require the use of congruent triangles. (Though it would not rule it out!) We have been developing the logical backbone of such a course and posting our work in progress here: http://www.mathedpage.org/transformations/

## ANSWERS AND COMMENTS

Here are the answers, along with ideas that could be included in a subsequent informal discussion. Note that we chose *inclusive* symmetry-based definitions. (For example, according to those definitions, an equilateral triangle is also isosceles; a rhombus is a special kind of kite; etc.)

1. *Isosceles triangle*. Since there are three vertices, the line of symmetry must pass through one of them, and the other two must be reflections of each other.

2. *Equilateral triangle*. If students try to create a triangle with exactly two lines of symmetry, they will find that it is not possible. If there are two, there must be three. You may ask what rotational symmetry is present (120° rotational symmetry.)

3. *Parallelogram*. This can be verified by tracing a tangram parallelogram, and rotating it 180°.

4. *Kite*. Some trial and error will show that the only way to have a line of symmetry that passes through vertices is to have it pass through opposite vertices.

5. *Isosceles Trapezoid*. In a quadrilateral, if a line of symmetry does not pass through vertices, it must pass through the midpoints of opposite sides. If there is only one such line, the sides it passes through are called *bases*, and the other two are called *legs*.

6. *Rhombus*. A rhombus is a special kite. It has 180° rotational symmetry, so it is also a special parallelogram.

7. *Rectangle*. A rectangle is a special isosceles trapezoid. It has 180° rotational symmetry, so it is also a special parallelogram.

8. *Square*. Given its symmetries, a square is a rhombus, a rectangle, and a parallelogram. It has 90° rotational symmetry.

9a. *Regular n-gon*. In this case, $n$ is even.

9b. *Regular n-gon*. In this case, $n$ is odd.

Pattern blocks can be used to illustrate the symmetries of #2, 5, 6, 8, and 9a.